Wolfgang Wildgen (University of Bremen, Germany)

René Thom's archetypal morphologies of human language and his semiophysics applied to visual art and music.¹

Abstract

René Thom made his major contributions to linguistics and semiotics, first, in his book "Stabilité Structurelle et Morphogenèse", in the chapter "L'homo loquens" (The Speaking Man) in 1972, and second, in his book "Esquisse d'une sémiophysique", 1988, chapter 8. In the first, he presented a list of 16 "morphologies archétypes"; in the second, he proposed to bridge the gap between the sciences of culture and communication and the natural sciences. The present contribution describes how Thom's linguistic conjectures and his program of "sémiophysique" have been further developed and modified in ongoing research until today. The last section exemplifies applying his topological and dynamic view to visual art and music.

Although a final evaluation of Thom's journey into linguistics and semiotics cannot be given, its impact on contemporary and future thought in linguistics and semiotics can be estimated.

1. **Introduction**

René Thom and his work have indirectly become part of my scientific biography. In 1977, I sought to extend (mathematized) linguistic theory to the fields of topology (abstract spatiality) and dynamics (language creation and development). It was rather by chance that I came across an English version of ""Stabilité Structurelle et Morphogenèse"" (Structural Stability and

¹ Contribution to the conference 20th to 22nd September 2023 at the IHES (Paris) on the occasion of the centenary of René 'Thom's birth. The present text contains changes and elaborations after the conference.

Morphogenesis) and bought the "second revised, corrected and enlarged French edition" from 1977. Like many readers, I was fascinated by the new perspectives and nevertheless disconcerted by the difficulty of reading. However, I decided to place Thom's questioning at the center of my Habilitation thesis entitled *Dynamics of communication* (germ. "Verständigungsdynamik").²

My first personal contact with René Thom was during the International Semiotics Congress in Vienna in 1979, where he gave a plenary lecture. I met him after his lecture, and he came to listen to my contribution on verbal valences from a catastrophist perspective. He invited me to come and listen to his lecture at the premises of the "Östereichische Studiengesellschaft für Kybernetik" (Austrian Cybernetics Society, Vienna). It was not until 1982, during the Cerisy colloquium "Logos et Catastrophes. À partir de l'oeuvre de René Thom", organized by Jean Petitot, that our paths crossed again. Finally, René Thom invited me to spend the months of November and December 1987 in Bures-sur-Yvette, where I rubbed shoulders with him daily.

The objective of my contribution is to summarize the research I conducted from 1977 to 2023 on the semiotic and linguistic work of René Thom.

2. René Thom's contribution to linguistics

In this book that brought René Thom admiration and international fame, only subchapter 13.4, p. 309-315, treats with language (under the title "Homo loquens"). The list of 16 archetypes discussed in this section stands on p.312. In a paper "Topologie et linguistique" published in "Essays on Topology and Related Topics" (1970), he gave a complete description of the underlying conjecture on basic schemata of sentences, a topic classically treated regarding logics and the notions of subject and predicate. His conjecture was a provocation for linguists standing

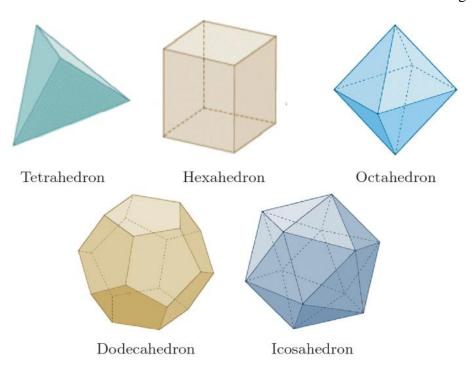
-

² A Habilitation grant (from the German research company, DFG), 1977-1979, allowed me to complete my knowledge of differential topology, helped by mathematics professors Klaus Jänich (Regensburg) and Bruno Kramm (Bayreuth). A short version in English was published in Wildgen (1982); major chapters (corrected and augmented) were published in Wildgen (1985; in German). A grammar based on the notions of morphogenesis and topology was put forward in Wildgen (1999; in French)

in the tradition of structuralism and Ferdinand de Saussure and those believing that logic (either traditional or modern) is the proper frame for questions of semantics and ontology.

Thom's innovative and radical thought has two main sources: On the one hand, his training and research in mathematics, especially in differential topology, and on the other hand, his philosophical reflection, which motivated him to consider the natural sciences (e.g., experimental physics, the ideal of modern sciences) and human sciences from a new perspective. His concepts for linguistics are only a tiny part of this new conception of scientific work.

For Thom, mathematics, and more particularly the tradition stemming from Poincaré having given significant results in the theory of singularities and structural stability, provides, as Kant already understood concerning Euclid's geometry, the proper conceptual frame that can make the mass of empirical observations or experiments understandable. His attitude may be compared to Plato's reconstruction of the universe and the human mind in his dialogue "Timaeus" ³.



³ His interlocutor, Timaeus presents in a kind of treatise the view of Greek mathematicians in Plato's time standing in the tradition of the mythical Pythagoras (cf. the comparison of platonic solids and catstrophes in Wildgen, 2020).

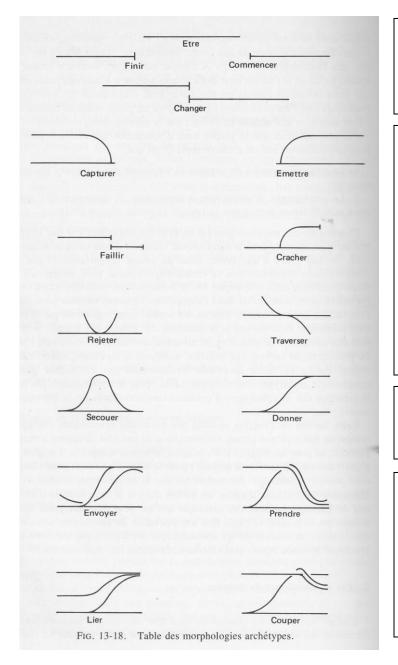
Figure 1 The five platonic solids (Springer Link)

In his book Structural Stability and Morphogenesis, Thom does not refer to any of his time's expert semioticians or linguists (in the late sixties). He refers only to Lucien Tesnière, whose main posthumous work was published in 1959 (but who had developed his innovative ideas in the 1930s). The list contains 16 verbs in French; in the perspective of the universal scope of Thom's conjecture, the proper basis should be all languages (currently one counts 6000 to 7000 languages). The lexicon of verbs in every single language amounts to hundreds if not thousands of verbs. therefore Thom's ist of French verbs can only be a suggestion for further work by linguists. Based on the state of the art in verbal semantics and linguistic typology, we propose to suggest to consider four different levels of application:

- 1. Types of temporal focus in verbs (and sentences), called traditionally "Aktionsarten", for instance: continuative / egressive / ingressive / resultative.
- 2. Valence of the verb (values 1 (0) to 3; in causative constructions 4 (cf. Tesnière, 1979). Beyond the chemical analogy, one could speak of verbal dynamics, i.e. a force profile, or with reference to case system an ergativity-scale. The central feature is the restriction to 3 (with extension) 4 categories; this is link to the limits of Platonic solids to five (to three if we consider two pairs of duals).
- 3. Beyond valence, manner specifications accompany the valence bound sentential kernel; further specification vary strongly with languages (and can be neglected in a model with universal scope)
- 4. Sentences are often organized along a topic profile. Thus the notion of Subject that was central for logical analysis since Aristotle belongs to the level of topic scales.⁴

-

⁴ The Object (direct and indirect in Tesnière's grammar) has a lower topicrank than the Subject.



Action type ("Aktionarten")marked morphologically or syntactically: continuative/egressive/ingressive/resultative a.o.

Capture and Emission are both central types of 2-valent verbs and corresponding constructions (transitive in nominative accusative case systems; cf. later)

Faillir (almost do) is a modal specification.

Cracher (spit), Rejeter (reject), Traverser (walk across), Secouer (shake) are verbs with an integrated manner specification of the type Capture / Emission.

Donner (give) demonstrates the type of 3-valent verbs and corresponding constructions

Envoyer (send) is a verb (construction) with valence 4. It implies a medium or an instrument. This is also the case for the other three verbs: Prendre (take) (with the hand), lier (ty) (with a rope), Couper (cut) (with a knife). These (adverbial) specifications are not strictly governed by general laws.

Figure 2 Thom's list of 16 archetypal morphologies basic for the semantics of verbs

As a result of this inspection of the examples given by René Thom we are left with univalent processes and their "Aktionsarten" (first field), the 2-valent types Capture and Emit, the 2-valent type Give and the 4-valent type Send. But the scope of topological semantics goes further; it advocates the existence of processual images as he content of basic verb types. This claim goes

beyond traditional semantics. The schools of Cognitive Semantics (Fillmore, Talmy, Lakoff, Langacker) made a similar claim but could not substantiate it because they were not able to integrate the geometrical/topological aspects into their modelling. Cf. th critique in Wildgen (1994: chapter 1) and the historiographical overview in Wildgen (2008: chapters 3 to 5) and (2010: chapter 12),

2.1. Thom's morpho-dynamic turn opposed to mainstream structuralism (1916 – present)

Ferdinand de Saussure (1857-1913), professor of comparative linguistics (mainly Indo-European languages) was at the origin of classical structuralism, whereas Charles S. Peirce (1839-1914), was the father of modern semiotics, a logician and chemist. He constructed semiotics on the basis of a relational and diagrammatic logic, he had devised. The mathematician René Thom, who started with applications in biology, did not share these traditions and made a fresh and independent start into semiotics and human communication.

In a talk in Geneva in 1970, René Thom refers to Saussure and outlines his model of "topological linguistics". For René Thom, the morphologies in the human sphere are linked to the morphologies found in animals and the nature surrounding animals. The visual, acoustic, olfactory, and tactile media of communication have their basis in the corporeality and vital functions of living beings, and this is also valid for language that is a late-comer from an evolutionary point of view.

The formal concepts René Thom puts in the foreground differ fundamentally from those advocated and used in linguistic structuralism. In his Theory of Cases (1935), Louis Hjelmslev, the leader of the Copenhagen school of structuralism, postulated a "prelogic", i.e. a kind of "§natural" logic underlying mathematical logics in the sense of the "Principia Mathematica" (Whitehead and Russel, 1910), It formed the background of the analysis of case structures, i.e., the typology and case marking of nouns dependent on a verb. On the other hand, Chomsky considered in 1957 the algebra of free groups (monoids) as an adequate background. Thom starts from a geometric point of view (more geometrico), generalized within the field of differential

topology. For René Thom, the spatial and temporal aspects (preconditions of a causal analysis) are primary. His idea of topological linguistics tries to integrate semiotics and other humanistic sciences into the general line of modern science, initiated around 1600 (Kepler and Galilei), brought to a first climax in the 17th century by Descartes, Leibniz, and Newton and responsible for the explosion of knowledge in the next centuries. His proposals ask for a reorientation of questions and scientific strategies in linguistics and an adaptation to the standards established for the other sciences inspired by the development of the natural sciences. It is, therefore, not surprising that linguistic and semiotic circles have neither understood nor accepted this radical innovation by René Thom.

In order to allow for an evaluation of Thom's conjecture on catastrophe theoretical (topological) semantics. I will sketch major traits using later elaborations in my scientific research and related elaborations by Jean Petitot. Cf. the monographs Wildgen (1982, 1985, 1994, 1999. and 2023) and Petitot (1985, 1992, and 2011).

2.2. Archetypal morphologies. A linguistic elaboration 5

In my book "Catastrophe Theoretical Semantics" (1982), the basis of applications was the list of simple catastrophes (Arnold, 1972), which contains the "elementary catastrophes" as a subfield (codimension 1 to 4).

⁵ This paragraph summarizes some contents of Wildgen (1982, 1985, 1994, and 2023, chapter 6).

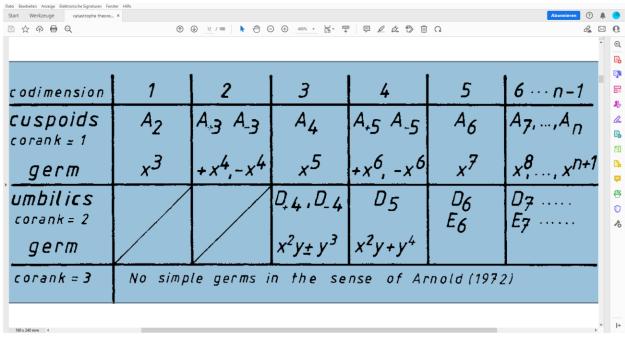


Figure 3 Simple germs in the sense of Arnold (1986) (partial picture); screenshot from Wildgen (1982:7)

The elaboration in my book considered the duals of the cuspoids (A-series) and neglected the restriction to codimension \leq 4. In the case of the umbilics Series D), the results of Godwin (1971) and Callahan (1978 and 1980) implied corrections on the four-valued archetypal morphologies. They had to be derived from the compactified umbilics (closing the "Thalweg") in the context of the "Exceptionals" E_6 , E_7 , E_8 , and the double cusp, X_9 . Only the "sending "morphology "("envoyer"", in Fig. 2) was derived explicitly from the compactified elliptic umbilic (D4); cf. Wildgen (1985: 204-208). In the following, we shall not discuss the mathematical details but the interpretation of these archetypes in the context of linguistic analysis.

2.3. Sketch of an empirical interpretation of Thom's conjecture

The category of verbs that logicians traditionally neglected ⁶ becomes the theoretical core of a morphogenetic analysis. Thom was motivated in his conjecture on basic sentence schemata by

⁶ See for example the logic of Port Royal which wanted to reduce this part of the lexicon to the single verb "to be".

the work of Lucien Tesnière (1893-1954), who had put the verb (and not the subject) at the center of syntactic analysis and who postulated a dynamic relationship between the verb and its dependent nominal syntactic complements (called "actants"; in French grammar: subject, direct and indirect complement). The term "valence", which Tesnière introduced, referred to chemical) structures, but he and his followers could not complete this reference to chemistry beyond a pedagogical metaphor.⁷

The first questions that the categorization of a process, an event, or action raises are:

- What structurally stable structures underlie the processes in question such that a lexicon of verbs can refer to them?
- What are the motor controls and perceptual patterns that mentally appropriate such processes?
- Finally, how are perceptual and motor controls correlated with the appropriate linguistic forms?

The motor programs modify the autonomous dynamics of the body's extremities and their contact with objects (e.g., the floor for the feet). These autonomous dynamic structures determine variables for perceiving movements and the cognition that regulates these systems. It can be concluded that the brain reflects the external dynamics (by adding other parameters and distorting them in its metric). However, the question remains: Does this coordination with external physics also control the higher levels of cognition, especially linguistic cognition? We assume an intermediate level, called imaginal or schematic. It applies processes that become increasingly independent from the psychophysical grounding and more context-dependent (as a consequence, they depend on chance). In what follows, we will start from the psychophysical level to find schematizations (imaginal representations) that underlie the semantics of verbs. We distinguish three typical levels of organization in the lexicon of verbs and verbal phrases:

• Locomotion and its linguistic schematization,

⁷ The link must be sought indirectly via Gibbs phase rule and Euler's formula for regular polyhedra. Cf. Wildgen (1994: 46f.). Peirce had around 1900 already proposed an analogy between chemical valences and basic sentence patterns. However, he argued in the context of structural chemistry and not modern nuclear chemistry.

- control of an object by an agent,
- interaction between agents.

In the case of the movement of the body's extremities (e.g., legs, arms), one can use the physical description of the pendulum (or the double pendulum) as a fundamental schema. Fig. 4 shows the correspondence between the double pendulum and the walking motion. The movement of the body supported by the hips is in coordination with the relative movements of the thigh (measured at the knee) and the lower leg (measured at the ankle):

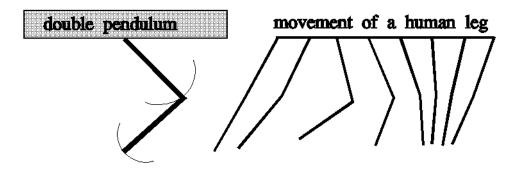


Fig. 4 The movements of the double pendulum and of a leg in walking

Specific movements are directed toward a target (an attractor). This orientation implies a separation of the starting and the ending points. This bimodality corresponds to a transition in space, and its lexical correlates are (in English) *enter/exit* or *come/leave*. The first dichotomy takes up the archetypal morphologies called *Begin* and *End* ("*finir*", *commencer*") in Thom's list. They concern the "Aktionsarten" of verbs and may be realized lexically, morphologically or syntactically (cf. the comments on Thom's list in figure 2). In the grammars of Greek, Russian and other Indo-European languages they were a major concern of classical grammarians.

In the control of an object by an agent, the configurational aspect and the energetic aspect, which introduces the force of the agent and the effect of this force on the object, can be distinguished. A basic type of effect is the creation, destruction, and regeneration of objects (elements of the environment). It is easy to see the correspondence of this type of action with the emission and capture schemata in Thom's list in Fig. 2 Capture and Emit ("capturer, émettre"). In the lexicon,

the corresponding verbs are, in most cases, divalent (for instance transitive in case systems of the type nominative-accusative (cf. later comments).

The process effected by an agent can also influence or alter the quality of an object. In this case, the background is not space/time and motion but a quality space (qualia). All languages provide a lexicon of entities related to bipolar quality scales. In Western languages, the lexicon of adjectives (and participles) demonstrates the field. They are further linked to verbs describing such changes:

Examples: *long – short*, *strong – weak*, *good – bad* or *dirty – clean*, *right – twisted*, and similar ones in other languages. Corresponding verbs of quality change are *shorten*, *strengthen*, *clean*, and *twist*.

Topologically, the phenomenon of bimodality as a stable pattern in the organization of the quality space is akin to Thom's elementary catastrophe cusp and the archetype called Change ("*changer*")), see the list in Figure 2 (above).⁸

An interaction scene that connects several human agents presupposes a very complex perceptual and conceptual analysis of the individual observing the scene. From a repertoire of action controls, one can reconstruct possible patterns of social interaction. However, it turns out that only a small group of these coordinated interactions achieve high stability, allowing schematization and semantic classification.

a) From the angle of the spatial configuration, we can describe the gift, i.e., the scene during which two people exchange an object, by the topology of the attentional focuses. Petitot (2011: 272ff) elaborated on a proposal by Sir Christopher Zeeman (1925 -2016) for a model that uses cognitive algorithms, such as cut locus and diffusion contours. In this elaboration toward neuro-vision, the catastrophe theoretical model gains more theoretical depth because it shows that the mathematics of differential topology can be used in the specific context of

11

⁸ Charles Osgood and his team have reconstructed statistically a three-dimensional space underlying hundreds of such quality scales in many languages (The theory is called "Semantic Differential", cf. Osgood, 1976).

visual scene analysis. The semantic archetype would, in this perspective, be first the result of morphogenesis in visual pattern recognition. Non-human primates and pre-human species (for instance, Australophicines) probably had this faculty. This pattern would have gained social and cultural significance in the transition to language.

b) The 4-valent archetype of Sending ("envoyer" and similar types with a mediating fourth attractor can be derived from the compactified elliptic umbilic (D4). Three "Thalweg", i.e. vectorfields towards the negative, can be closed in the context of the embedding Exceptinal E₆ (cf. Wildgen, 1985: 204-212).

We shall only show the derivation of the archetype of *Giving*, that may stand for the class of 3-valent processes and actions that involve a transitory object, quality or instrument, and the archetype of *Sending*, that may stand also for other instrumentally or causally more complex types of processes and actions. The heavy lines stand for the attractors: Peter, hammer, and Anne, the encircled are bifurcation, where the middle attractor is emitted, gains dominance and is captured or eliminated, the arrows point to dominance shifts in the neighborhood of an unstable equality.⁹

_

⁹ The sociologist Marcel Mauss (1872-1950) made the "gift exchange" or ""ceremonial exchange"" a pillar of his theory of society.

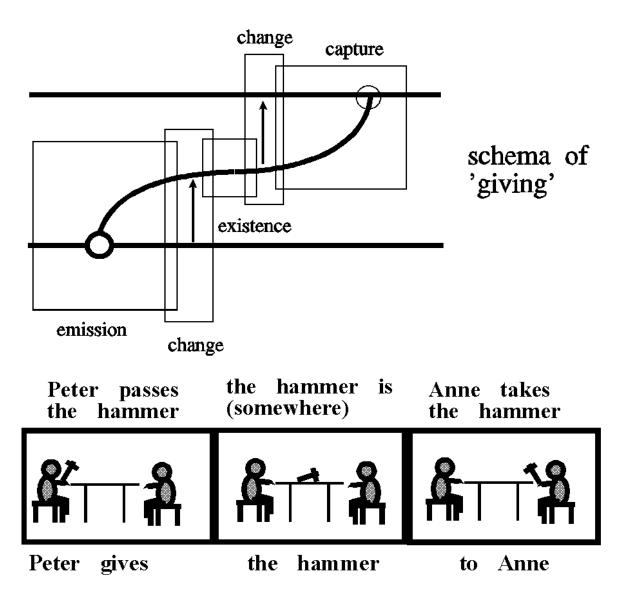


Figure 5 The schema of Giving derived from the butterfly (A_6) and comic-strip as representation of the sentence: Peter gives the hammer to Anne (cf. Wildgen, 1994: 146)

The derivation of the archetype of Sending (and other 4-valence processes) and actions is based on the elliptic umbilic that has corankk 2 (two internal variables x and y), an atractor on one side and an repelor on the other. The attractor is surrounded by saddles that lead to three valleys of the vector field (called Thalweg). If they are closed (compactified), one obtains three further attractors. A process of the type Giving may link these attractors as in the case of simpler archetype of Giving, while the central attractor controls or mediates the process.

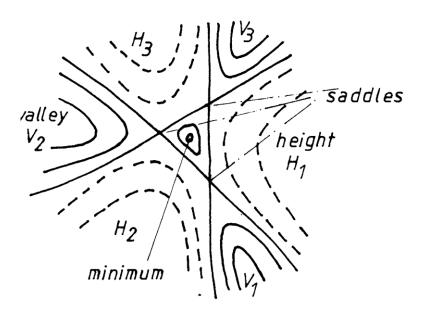


Figure 6 The vector field around the central minimum in the elliptic umbilic, the saddles, and the valleys V1, V2, and V3 that can be closed by compactification and are the basis of the 4-valence process called Sending

2.4. The validity of Thom's conjecture after 50 years (1972-2022)

Evaluating Thom's conjecture on verbal valences (deep cases) and basic sentence constructions after 50 years is difficult for several reasons, although several technical (mathematical) defaults could be enhanced (cf. section 2.1). First, in the lexicon of verbs, there are different layers of interpretation, also called local ontologies, from spatial motion, action, and (social) interaction to qualitative changes, and to mental processes (virtual ontologies). Second, the case patterns in classical Indo-European languages, e.g., Sanskrit, Greek, and Latin, do not represent the corpus of languages worldwide. There are languages without case classification and marking. Word order, stress patterns, or contextual cues may provide comparable information (e.g. Chinese). One distinguishes two basic types in those languages with explicit case marking: nominative-accusative and ergative-absolutive. Many Western languages prefer the accusative-nominative type (exception the Basque language), but case marking with flectional suffixes has disappeared in many language e.g. in Romance languages derived from Latin and in some Germanic languages (e.g. English). Language change the type of realization and transitory states can be

observed. This is the topic on the theory of grammaticalization. Contrary to de Saussure's preeminence of synchronic analysis, such processes ask for a diachronic access. In this case the dynamic models proposed by Thom in 1972 gains a new level of relevance, insofar as it can describe basic time-dependent processes in language change as the unfolding of underlying principles in historical time (or in spontaneous creativity at the micro level of language change)

The author proposed an catastrophe theoretical model of this situation based on a set of paths in the bifurcation plane of the cusp (A_4) . Cf. Wildgen (2017).

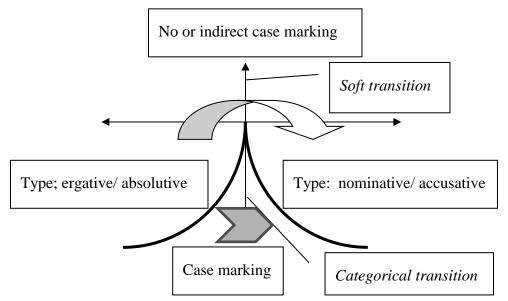


Figure 7 A "cusp" - model of the typology of case marking and underlying historical processes

In summary, empirical research could only partially match Thom's conjecture with facts in the world's languages. However, his hypothesis opened a window to understanding the morphogenesis of case marking and "meaning building" in sentences. Moreover, it introduced a spatial and dynamical representation of case concepts that are cognitively more realistic than representations in terms of predicate calculus that dominated formal semantics in the 20^{th} century. 10

¹⁰ The proposals of quasi-dynamic gestalts in Cognitive Semantics in the style of Lakoff, Langacker, and TaLmy has been criticized in Wildgen (1994: chapter 1) as a purely didactic short hand of intuitions in early Gestalt theory (e.g. Kurt Lewin's topological psychology, 1936).

15

3. The semiophysics of Thom (1988) and its application to art and music

A second phase of Thom's research in semiotics and linguistics began around 1978 and led to his book "Sketch of a Semiophysics" (Thom, 1988). He generalized the notions of salience ("saillance") and pregnancy/vital relevance ("prégnance")¹¹ to circumscribe the forces at work in the creation and stable use of semiotic patterns. Salience relates to the sense organs (sight, hearing, smell, touch, and others) and the affordances in the surrounding world. Pregnancy considers the biological and mental functions of action and communication linked to perceiving salient features and perceptually controlled action. The coordination of both enables "meaning building", i.e., the morphogenesis of meaning. In the case of language, it surfaces on very different temporal and spatial scales:

- The origin of language,
- Language change,
- Pidgin or Creole language, they witness the capacity to form spontaneous linguistic patterns.
- The infant's discovery of lexica meaning and syntax,
- Lexical and syntactic innovation in poetic creativity.

Semiophysics builds a bridge between the traditionally separated fields of humanistic and social sciences and the fields of the natural sciences. The mathematics of "dynamic systems theory" (catastrophe theory, synergetics, chaos theory, stochastic dynamic, and other subfields) may serve as a catalytic converter of new ideas linking both sides. Thus, semiophysics is rather a program of research and not yet an established discipline. In the following, we shall consider with priority applications of semiophysics beyond language, i.e., visual and musical communication. ¹²

-

¹¹ His notions are similar to those in ethology (Konrad Lorenz) and Gestalt theory (valence, symbolic pregnancy); cf. Wildgen (2010a)

¹² The psychophysical resources are clearer in the visual domain (e.g. art) and the domain of music (hearing and performing music). In the case of language, the "arbitraire du signe" (arbitrariness of the correlation of sign shape and meaning) emphasized by de Saussure and linguistic structuralism has the effect of a stochastic noise masking the operation of basic principles founded in the nature of the human environment and the human body. In the following

3.1. The case of visual art

Leonardo da Vinci (1452-1519) was a new type of artist in Renaissance times connected to contemporary science and philosophy. He illustrated Luca Pacioli's treatise "De divina proportione" (composed 1497, published 1509) and devised the first skeletal representation of a platonic solid.

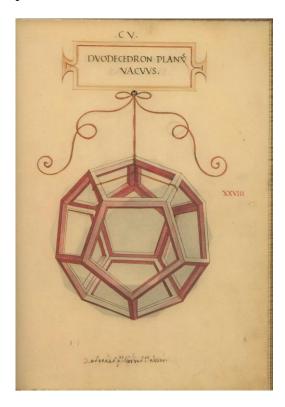


Figure 8 Leonardo's picture of the (regular) dodecahedron in the treatise "De divina Proportine" by Luca Pacioli (printed 1509)

The following section discusses dynamic and topological features in Leonardo's mural "Last Supper" (1495-1498) as an example of semiophysics in art.

examples we have chosen cases where cultural (or academic) traditions are less prominent and the creativity of the artist in his confrontation with the human mind/body and nature is prominent.

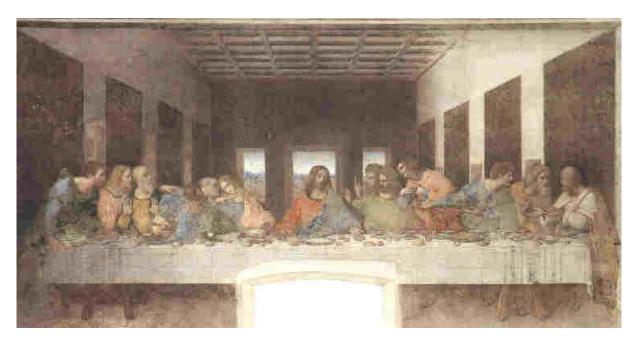


Fig. 9: Leonardo da Vinci The Lord's Supper in Milan, Santa Maria Delle Grazie (1495-1498) (Wikipedia Commons)

In Leonardo's mural "Last Supper", the central figure, Jesus, represents the type of an equilateral triangle (in three dimensions, a tetrahedron). The refectory is a box similar in shape to a cube or if one follows the lines of perspective one half of the octahedron. The four groups of apostles also have a triangular shape, but they are agitated by a perturbation with its origin in Jesus saying, "One of you will betray me". The group with John, Judas, and Peter to the left of Jesus is singular. John is leaning back to listen to Peter, whereas Judas resists the shock wave issued by Jesus (Peter points with the knob of his knife to Judas). Leonardo uses the mathematics of regular polygons and polyhedra to organize the traditional topic of "The Last Supper". As Slodowy (1988) showed, there is a link between regular polygons and the cuspoids and of regular polyhedra and the unfoldings E6, E7, and E8: the umbilics are the topological correspondent to the intermediate Kleinian Dihedra.

For Leonardo, natural dynamics is the source of bodily dynamics and, finally, the dynamics of the mind. Therefore, his art starts from the physics of objects and bodies in space/time, resulting in a morphogenesis of meaning in a semiotic/symbolic space.

The dynamic of traditions in art are dominated by repetitions with deformations. This copy feature in work of art can be demonstrated for the series of paintings with the topic "Last Supper since the 11th century (cf. Wildgen, 2004 and 2010). Elements of Leonardo's mural reappear in modern paintings. For instance, Andy Warhol (1987) reduced the colors and reassembled subfields in Leonardo's painting in a collage. Here the figurative reference to the biblical text is only a background, the immediate reference is to Leonardo's mural, but instead of making a copy with variations, Warhol selects major motifs, reduces the colors and the magnitude.

Mathematically the picture is a semi-periodic tessellation of the rectangular surface with partial pictures extracted from Leonardo's mural. and Cf. Wildgen (2018: 86-89).



Figure 10 Andy Warhol, 1987, Last Supper (regarding Leonardo's mural in Milan)

Concerning the mathematics of non-linear and chaotic dynamics, Pollock's action paintings correspond to new geometrical and dynamic principles, as did Leonardo's painting, relative to the mathematics of the 16th century. Jackson Pollock (1912-1956) leaves the static configuration of the first abstract paintings (e.g., Kandinsky "Komposition" VII, 1913) and makes a radical move towards motion and action in paintings. In his dipping technique, colour flows out of containers or is sprayed over the canvas. If we look at Pollock's painting "Grey and Red" (1948), we can understand the thin threads as the trace of quick motion and the thick lines or knots as halting points (assuming a constant paint flow).

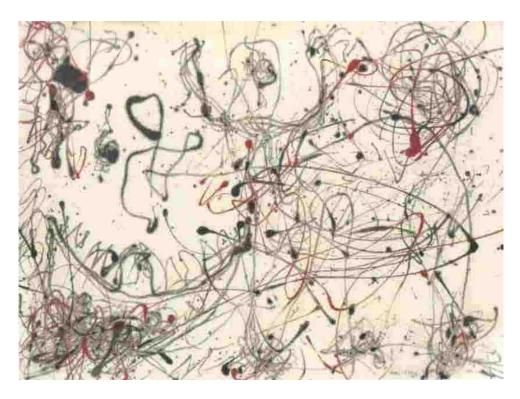


Figure 11: "Grey and Red", 1948, 57,5 x 78,4 cm, Frederik R. Weisman Foundation, Los Angeles

Basic Gestalt laws of composition in paintings, e.g., up and down, left and right, or diagonals linking the corners, are disregarded. Taylor, Nicolich, and Jonas (1999) showed that Pollock's paintings have a fractal order. Specifically, two fractal measures are characteristic: The first is related to rapid hand motion on a scale of 2.5 to 5 cm, and the second scale (5 cm to 2.5 m) corresponds to his arm and body motion (into and around the canvas). Moreover, the fractal dimension increased when he developed his new painting style (*action painting*). In 1943, it was still linear (dimension 1). Almost ten years later, the fractal dimension is 1,72, thus nearer to a line filling the given surface. The principles of visual salience ("saillance" in Thom, 1988) exploit the psycho-physical potential f humans, although the profile and preferences in this use

_

¹³ The measurement uses the box counting technique. Boxes (i.e. quadratic surfaces) on the canvas are made smaller and smaller and one calculates how many steps of reduction are necessary to make the distinction between line and void surface disappear. If it does not disappear (no void) the dimension is 2, i.e., the plane is *full*.

change historically. The dimension of pregnancy (meaning, "prégnance" in Thom, 1988) changes also. If in the Middle Ages, religious texts gave the preferred contents artist had to represent, mythological contexts came to the fore in Renaissance. Eventually the court-life and later everyday life in the civic society, industrial contexts and the privacy of the artists became prominent. In abstract art, all these variable contexts of content in art were eliminated; art began to visualize art ("l'art pour l'art).

3.2. The semiophysics of music

The physics (acoustics) of overtones in the human voice and musical instruments is the essential pillar of human perception and control of musical forms. The overtones have a regular physiological equivalent in the inner ear (cochlea). At the same time, this natural grid can be stretched in time, i.e., the resonating overtones can be "spelled out" as elements of a melody. As a result, the sequential order in music inherits the natural ordering system of the overtones. ¹⁴ Memory effects impose the first severe restriction because memory works like a hedge in the cerebral recognition and assembling of complexes of musical signs.

As a consequence of these conditions of sound perception in humans emerges the phenomenon of categorical perception, i.e., the transitions from continuous scales (in physics) to boundaries imposed on this continuum, together with a radical reduction of the space of choices in musical perception. Eventually, the instruments devised and used by a population and later written music codes led to a wealth of musical forms and "meanings".

The human gait in walking and running is a major source of rhythm in music. Walking and dancing are subject to coordination dynamics, i.e., the motion of the right and the left leg are coordinated, and the motion of one dancer is coordinated with the motion of other dancers. The gesture is a particular form of movement that emerges from the field of physical movement and contains an intentional momentum, such as nodding, pointing, or eye-gazing. In music, gestures

21

¹⁴ The space of possible auditory distinctions is vast at this first level. Sundberg (1991: 62) computes a maximum of 1400 pitch distinctions and 280 distinctions of loudness.

are a basic type of musical gestalt. A melody may be decomposed into several gestures, some of which are within the boundaries of beats, some exceeding them. Thus, musical communication can be understood to emerge from the physics/physiology of hearing, the motor dynamics of limbs in locomotion and dance, and the articulatory dynamics in singing. The handling of musical instruments is a derivative of these dynamic sources. In the history of musical theory, mathematicians since the legendary Pythagoras, later Leonard Euler, and more recently Mazzola (2202) propose a pertinent space in which music is embedded.

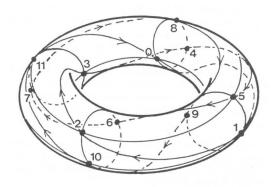


Figure 12 The torus of tierces in Mazzola (2002). The twelve tones (0...11) are arranged according to the distance between a major third (meridian circles), e.g. (2), (6), (10) and a minor third (equatorial circles), e.g. (2), (5), (8), (11). The spiral movement on the torus (see the arrows) corresponds to the steps in seconds, i.e. the sequence of 12 semitones

Music theorists in different phases proposed practical rule systems for music composition (for instance, the rules of counterpoint in the 17th century, the paradigm of classical tonality, and atonal music in the 19th and 20th centuries). We take the simple case of a song and its melody. The course of the melody has a center that can be used as a zero expectation of melody progression, and the melody is the deformation of this standard. Leyton (2001) assumes a "memory" as the specific meaning of a melody. It contains information about forces that have become effective and influenced the melody's design. The forces that shape the melody push the melodic line up or down concerning the middle (comfortable) level. These force-patterns and their memory traces are natural candidates of musical pregnancy (meaning), that is not dependent on literary or other figural attributions.

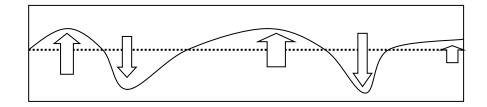


Fig. 13 The melody course as a result of forces acting on a medium level

These movements are perceived as meaningful sign shapes g. Downward movement may, e.g., express sadness or resignation; an ascending movement, on the other hand, enthusiasm or hope. We take as an example the song "Hallelujah" by Leonard Cohen. It moves in a flat curve around G/A in the first ten beats. After "It goes like this", it climbs up in steps until it reaches the E'. The repeated "Hallelujah" descends in a wave-like movement until it reaches the lower and final C, the resting point (the song is written in C).

If we apply Leyton's concept of memory of forces, we can draw the following schema of applied forces (Fig. 14) (Video link: http://www.leonardcohen.com/).

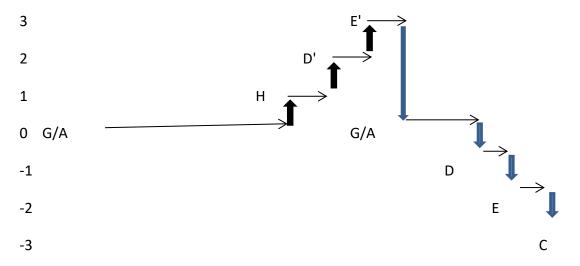


Fig. 14 Motion and force in the song "Hallelujah" by Leonard Cohen

Mathematical considerations were already central to musical thinking and composing in antiquity. In the 20th century, with synthesizers and computer-aided music, mathematically sophisticated types of music became typical in the avant-garde scene. Muso-mathematical

considerations have played a role in musical composition since Claude Debussy (1862-1918) and Olivier Messiaen (1908-1992). The topological network of tunes was discovered at the end of the 19th century and gave rise to the "Tonnetz"-architectures of music based on mathematical group theory. The next generations of composers took up the suggestions of Thom's morphogenesis (cf. Andreatta, 2018b) and the concepts introduced by Chaos Theory and Fractal Geometry. Pagano (2018) analyzed Ligeti's piano studies: Vertigo, L'Escalier du diable, and Columnia Infinita (written around 1991). She writes: "The Devil's Staircase is a mathematical function, and the piece effectively describes it" (ibid. 96).

These hints may be sufficient to show that mathematical models of music are not only a possible means to understand music; musicians also use them to compose music and open new musical performance and understanding fields.

4. The relevance of Thom's linguistic work for the future development of linguistics

The work of René Thom figures like a rocky island in this tide of innovations in linguistics. In the 1960s, his correspondence with biologist Conrad Hal Waddington (1905-1975) finally sparked his publications on morphogenesis (Thom, 1972). His contributions to differential topology (Fields Medal, 1958) enabled him to propose a new modeling method, "modo geometrico", in life and the human sciences. This approach crossed the barriers between the natural sciences (especially physics and chemistry), the life sciences (biology and medicine), and the human sciences. Modern mathematical models, especially the dynamical systems theory, served as a conceptual bridge between these strictly separated domains. Unfortunately, his program remained largely in draft form, the techniques of observation or experimentation in

-

¹⁵ Physiological processes, such as the heartbeat, seem to be regular, but they show fractal patterns Therefore it is not surprising that fractal patterns also are present in music. A team from the Max Plank Institute for Dynamics and Self-Organization in Göttingen, for example, discovered fractal patterns and self-similarity in the music of a well-known drummer (Jeff Porcaro). The fractal character arises when small deviations are generated and multiply in an iterative process, spread out, and finally overshoot the original pattern.

linguistics and semiotics were barely developed, and the statistical evaluation of the models proposed was course or inexistent. However, Thom's conjectures and sketches of models have opened up a vast field of theorizing and empirical approaches. The rapid development of the mathematics concerned (see chaotic, fractal, and stochastic dynamics), the turn of cognitive sciences towards naturalistic modeling (neuronal or biological), the expansion of semiotics in zoo-semiotics and inter- and intracellular communication prepared the ground for a generalization of the morpho-dynamic paradigm.

Bibliography

Arnheim, R., 1988: The Power of the Center: A Study of Composition in the Visual Arts. Berkeley: University of California Press.

Arnold, V.I., 1972. Normal Forms for Functions near Degenerate Critical Points, the Weyl Groups of A_k , D_k , and E_k , and Lagrangean Singularities. *Functional Analysis and its*Applications 6: 254-272.

Callahan, J.J., 1978. The Double Cusp has Five Minima. Math. Proc. Camb. Phil. Soc. 84: 537f.

Callahan, J.J., 1980. Bifurcation Geometry of E6. Mathematical Modelling 1: 283-309.

Chomsky, Noam, 1957, Syntactic Structures, Den Haag, Mouton.

Godwin, A.N.,1971. Two Dimensional Pictures of Thom's Parabolic Umbilic, *Inst.Hautes Études Sci.Math.*, 40: 117-138.

Hjelmslev Louis, [1935], 1972, *La catégorie des cas. Étude de grammaire générale*, I and II, réimprimé en 1972, Munich, Fink.

Mazzola, Guerino, 2002. The Topos of Music. Geometric Logic of Con- cepts, Theory and Performance, Birkhäuser, Basel.

Petitot, Jean, 1983, Théorie des catastrophes et structures sémio-narratives, *Actes sémiotiques*, V, 47-48, pp. 5-37.

Petitot, Jean, 1985, *Morphogenèse du sens. Pour un schématisme de la structure*, Paris, Presses universitaires de France.

Petitot, Jean (ed.), 1988, Logos et théorie des catastrophes. À partir de l'œuvre de René Thom, Genève, Editions Patiño

Petitot, Jean, 1992, *Physique du Sens*, Paris, Presses du CNRS.

Petitot, Jean, 2011. Cognitive Morphodynamics. Dynamical Morphological Models of Constituency in Perception and Syntax, Bern, Peter Lang.

Thom, René, 1970, Topologie et linguistique, in *Essays in Topology and Related Topics*, Haefliger A. and R. Narasinkan (ed.), Berlin, Springer, pp. 226-248.

Sundberg, J., 1991. The Science of MusicalSounds. AcademicPress, SanDiego.

Thom, René, [1972] 1977, *Stabilité structurelle et morphogenèse*, Paris, Interéditions (deuxième éd. revue, corrigée et augmentée, 1977).

Thom, René, 1988, Esquisse d'une Sémiophysique, Paris, InterEditions.

Thom, René, 1991, Saillance et Prégnance, *L'inconscient et la Science*, R. Dorey (éd,), Paris, Dunod, pp. 64 – 82.

Waddington, C.H., [1951] 1968, The Character of Biological Form, *Aspects of Form. A Symposium on Form in Nature and Art*, Lancelot Law Whyte (ed.) (2nd ed., 1968).

Wildgen, Wolfgang, 1982, Catastrophe Theoretical Semantics. An Elaboration and Application of René 'Thom's Theory, Amsterdam, Benjamins.

Wildgen, Wolfgang, 1985. Archetypensemantik. Grundlagen für eine dynamische Semantik auf der Basis der Katastrophentheorie, Tübingen, Narr (partial publication of the thesis submitted in 1979).

Wildgen, Wolfgang, 1994, *Process, Image, and Meaning. A Realistic Model of the Meanings of Sentences and Narrative Texts*, Amsterdam, Benjamins.

Wildgen, Wolfgang, 1999, De la grammaire au discours. Une approche morphodynamique, Bern, Lang.

Wildgen, Wolfgang, 2001, Kurt Lewin and the Rise of "Cognitive Sciences" in Germany: Cassirer, Bühler, Reichenbach », *The Dawn of Cognitive Science. Early European Contributors*, Albertazzi Liliana (ed.), Dordrecht, Kluwer, pp. 299-332.

Wildgen, Wolfgang, 2010a, Thom's Theory of "saillance" and "prégnance" and Modern Evolutionary Linguistics, *Semiosis and Catastrophes. René Thom's Semiotic Heritage*, Wolfgang Wildgen, Wolfgang and Per Aage Brandt (ed.), Bern, Lang, pp. 79-100.

Wildgen, Wolfgang, 2013, Visuelle Semiotik. Die Entfaltung des Sichtbaren. Vom Höhlenbild bis zur modernen Stadt, Bielefeld, transcript.

Wildgen, Wolfgang, 2017, En cas de catastrophe. Les systèmes casuels et la dynamique qualitative, Contribution au Colloque Petitot, Paris-Nanterre, 29 mai 2015, *Estudos Semióticos*, nº 13 (1), pp. 1-15, <u>online</u>.

Wildgen, Wolfgang, 2020. Structures, Archetypes, and Symbolic Forms. Applied Mathematics in Linguistics and Semiotics, in: Alberto Peruzzi and Silvano Zipoli Caiani (eds.), 2020. *Structures Mères. Semantics, Mathematics and Cognitive Sciences*, Springer, Cham: 165-185.internet.