Towards a Unifying Framework for Uncertainty in CPS

for Jan Peleska

Jim Woodcock

University of York | Aarhus University

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Outline

Introduction

Motivation

Prism’s Semantics

Probabilistic Predicative Programming

Conclusions

Jan Peleska

- This talk is dedicated with affection to Jan Peleska on his 65th birthday.
- We discuss a unifying theory of uncertainty in robotics and CPS.
- We use Hoare & He's UTP and Hehner's probabilistic predicative programming.
- This is a long-term research agenda at York and Aarhus universities.
- We start with a semantics for Prism and end with many questions.

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Uncertainty in Robotics

- Autonomous vehicle tries to pass quickly through intersection without signals.
- Counterintuitively, the vehicle slows down instead of accelerating.
- It gathers information on the intentions of pedestrians and other vehicles.
- This information helps the vehicle coordinate its actions with others.
- It achieves its overall goal faster in the long term.

- Robot manipulator tries to push an irregular object to a designated pose.
- The robot must minimise the number of actions.
- It decides not to push the object directly towards the final pose.
- It uses the first pushes to gather information on the object's centre of mass.
- The later pushes are now much more effective.

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- pGCL, MDPs, POMDPs, dynamic epistemic logic, epistemic mu-calculus, ...
- What would a unifying theory for uncertainty look like?

- Research Hypothesis

  We can unify different theories of uncertainty using:

  UTP probabilistic relations. Bayesian semantics. Information theory.

- We focus on a specific domain initially: robot planning.

  Modelling and solving robot decision and control tasks under uncertainty.

  Noisy sensing, imperfect control, environment changes, inaccurate models.

  Localisation and navigation, search and tracking, autonomous driving.

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- pGCL, MDPs, POMDPs, dynamic epistemic logic, epistemic mu-calculus, . . .
- What would a unifying theory for uncertainty look like?

- Research Hypothesis

  We can unify different theories of uncertainty using:

  UTP probabilistic relations. Bayesian semantics. Information theory.

- We focus on a specific domain initially: robot planning.

  Modelling and solving robot decision and control tasks under uncertainty.

  Noisy sensing, imperfect control, environment changes, inaccurate models.

  Localisation and navigation, search and tracking, autonomous driving.

  Multi-robot systems, object manipulation, human-robot interaction.

- Robot reasons about outcomes of actions with limited sensor information.
- Actions have short-term rewards and inform long-term success.

Unifying Semantics for Prism

- Towards a specification-oriented semantics for proof and refinement.
- Start with DTMCs. Extend to MDPs, POMDPs, CTMCs, PAs, PTAs, POPTAs.
- Unifying semantics Powerful enough for SotA modelling languages.
- Denotational semantics Gold standard.
- Operational semantics Soundness wrto denotational semantics.
- Algebraic semantics Derived from opsem soundness proof.
- Programming logic Probabilistic Hoare logic (cf. Hartog & de Vink).
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- Discussion on why we need a formal semantics for Prism.
- Nonprobabilistic semantics: Unity and Kripke semantics.
- Existing system module semantics for Prism.
- Predicative Programming technique as a Kripke semantics.
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Prism DTMC Example

- Throw a pair of six-sided dice until they are equal. How long will this take?

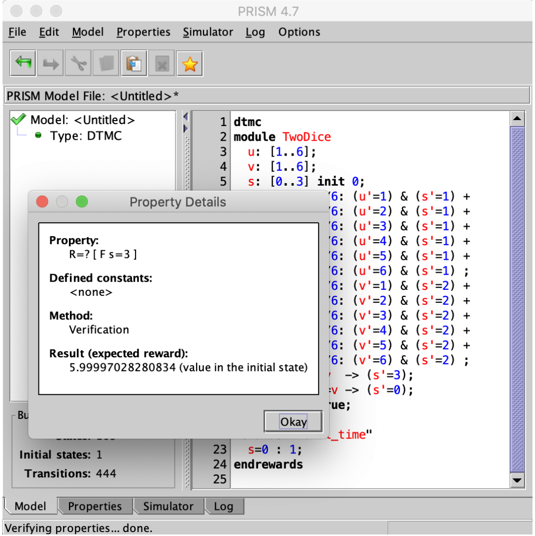
```
dtmc

module TwoDice
  u: [1..6];
  v: [1..6];
  s: [0..3] init 0;
  [] s=0 -> 1/6: (u'=1) & (s'=1) + 1/6: (u'=2) & (s'=1) + 1/6: (u'=3) & (s'=1) +
            1/6: (u'=4) & (s'=1) + 1/6: (u'=5) & (s'=1) + 1/6: (u'=6) & (s'=1) ;
  [] s=1 -> 1/6: (v'=1) & (s'=2) + 1/6: (v'=2) & (s'=2) + 1/6: (v'=3) & (s'=2) +
            1/6: (v'=4) & (s'=2) + 1/6: (v'=5) & (s'=2) + 1/6: (v'=6) & (s'=2) ;
  [] s=2 & u=v  -> (s'=3);
  [] s=2 & u!=v -> (s'=0);
  [] s=3 -> true;
endmodule

rewards "total_time"
  s=0 : 1;
endrewards
```

Prism Check

- We have a Prism model.
- But what properties does it have?
- How many throws of the dice-pair?
- How many throws, on average, do we need to terminate?
- Reward structure gives time steps.
- What's the expected time taken to reach, from the initial state, `s=3`?
- Prism says: you need `5.99997028280834` throws.
- But what if we have 10 dice?
- How many throws do we now need?



PRISM Model File: <Untitled>*

Model: <Untitled>
Type: DTMC

```
dtmc
module TwoDice
  u: [1..6];
  v: [1..6];
  s: [0..3] init 0;
                 6: (u'=1) & (s'=1) +
                 6: (u'=2) & (s'=1) +
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                 6: (u'=6) & (s'=1) ;
                 6: (v'=1) & (s'=2) +
                 6: (v'=2) & (s'=2) +
                 6: (v'=3) & (s'=2) +
                 6: (v'=4) & (s'=2) +
                 6: (v'=5) & (s'=2) +
                 6: (v'=6) & (s'=2) ;
                   -> (s'=3);
                 v -> (s'=0);
                 ue;

                 _time"
  s=0 : 1;
endrewards
```

Property Details

Property:
R=? [F s=3]

Defined constants:
<none>

Method:
Verification

Result (expected reward):
5.99997028280834 (value in the initial state)

Bu

Initial states: 1

Transitions: 444

Verifying properties... done.

Why Do We Need another Formal Semantics for Prism?

Prism's process algebra operators

- CSP-based. But some aspects are only syntactic, not semantic.
- Prism action labels are not CSP Events.
- Prism deadlock is not CSP Deadlock.
- Prism hiding is not CSP Hiding.

More powerful verification and validation of probabilistic systems

- Refinement Theory Correctness by construction.
- Assertions Theorem proving. Design by contract. Runtime checking.
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- Testing Theory (Probabilistic) testing can be formal, too.

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- Describes models with propositionally labelled states.
- Temporal logic semantics traditionally given using Kripke structures.
- Structure consists principally of a transition relation.
- Nodes represent reachable system states. Edges represent state transitions.
- Labelling function maps a node to a set of properties holding in that state.
- Why use Kripke structures?

  They represent closed finite-state models with independent state encoding.
- This captures the notion of observability to relate to actual executions.
- An observer might not be able to read all state variables.
- Trace: sequence of observable parts of states.

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Unity Single Command Semantics

- Prism $\setminus$ Probability $=$ Unity [Chandy & Misra]
- Guarded command $c \in C$ `[]g->u.`
- Sub-state space $S_c = \{ s \in S \mid s \models g \}$ UTP: this is simply g.
- Guard g predicate over variables in V.
- Command u assignments to variables in V.
- Update u of c $u : S_c \to S$.
- Example `[] x>y -> (x'=y) & (y'=x);`
- Semantics $x > y \wedge (x' = y) \wedge (y' = x) \wedge (z' = z)$.
- Extension variables (unmentioned) stay the same: Example `(z'=z)`.

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- First, define the matrix, for any $s, t \in S$:

$$\overline{P}(s,t) \;\widehat{=}\; \sum c : C \bullet \mu_{c,s}(t)$$

- The rows of $\overline{P}$ may sum to more than 1. Why?
- Local nondeterminism in a module: overlapping guards.
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  $$\overline{P}(s,t) \;\widehat{=}\; \sum c : C \bullet \mu_{c,s}(t)$$

- The rows of $\overline{P}$ may sum to more than 1. Why?
- Local nondeterminism in a module: overlapping guards.
- Prism displays a warning when local nondeterminism is detected in a DTMC.
- Nondeterministic choice is randomised.
- A probability distribution is obtained by normalising $\overline{P}$:

  $$P(s,t) \;\widehat{=}\; \overline{P}(s,t) / \sum u : S \bullet \overline{P}(s,u)$$

- Replaces nondeterminism by uniform probabilistic choice between transitions.

Outline

Introduction

Motivation

Prism's Semantics

Probabilistic Predicative Programming

Conclusions

Discrete Distribution

- Suppose that e is an expression with free variables v.
- Expression e is a discrete distribution if it satisfies two criteria:

  1. Its value (for all assignments to v) is a probability: $[0 \le e \le 1]$.
  2. Its sum (for all assignments to v) is 1: $\sum v \bullet e = 1$.

- Suppose n and m are strictly positive integers.
- Then $(1/2)^{n+m}$ is a distribution because it satisfies the two criteria:

  1. Values: $\forall n, m : 1 .. \infty \bullet 0 \le (1/2)^{n+m} \le 1$
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- Suppose n and m are nonnegative integers (in contrast to the last example).
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- Example 2^{-n}: says n has value 3 ⅛ of the time.
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- If $n, m : \mathbb{N}_1$ are distributed as $(1/2)^{n+m}$, then

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- Average value of $n - m$ as n and m vary over $\mathbb{N}_1$ with distribution $(1/2)^{n+m}$ is
  $$\sum n, m : \mathbb{N}_1 \bullet (n - m) * (1/2)^{n+m} \;=\; 0.$$

Normalisation

Definition (Normalisation)

If E's variables are n and m, then

$$\mathbf{N}\,(E) \;\widehat{=}\; E \,/\, \left(\sum n, m \bullet E\right)$$

- Let E be an expression:
  - Whose value (for all assignments of values) is nonnegative.
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$$[\neg P] = 1 - [P]$$

$$[k \in A] + [k \in B] = [k \in A \cup B] + [k \in A \cap B]$$

$$[x \in A \cap B] = [x \in A] * [x \in B]$$

$$[\, \forall m \bullet P(k,m)\,] = \prod m \bullet [P(k,m)]$$

$$[\, \exists m \bullet P(k,m)\,] = \min\{1, \sum m \bullet [P(k,m)]\}$$

$$\#\{\, m \mid P(k,m)\,\} = \sum m \bullet [P(k,m)]$$

Iverson Laws

$$\langle N \rangle = N > 0$$

$$\langle 1 \rangle = \textbf{true}$$

$$\langle 0 \rangle = \textbf{false}$$

$$[\, N \leq [\, \langle N \rangle \,] \,]$$

$$\langle [\, P \,] \rangle \sqsupseteq P$$

$$[\, \neg \langle N \rangle \,] = [\, N = 0 \,]$$

$$P \sqsupseteq Q \Rightarrow [\, P \,] \leq [\, Q \,]$$

$$M \leq N \Rightarrow \langle M \rangle \sqsupseteq \langle N \rangle$$

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Example: Killer Robots

- cyberman and the dalek attack the Tardis daily.
- *cyber* has probability $1/2$ of success.
- *dalek* has probability $3/10$ of success.
- *cyber* attacks with probability of $3/5$.
- *dalek* attacks with probability of $2/5$.
- What is the probability of a successful attack?
- Conditional probability: $\mathrm{P}(A \wedge B) = \mathrm{P}(A) * \mathrm{P}(B \mid A)$.

$$\mathrm{P}(\mathit{cyber}) = 3/5, \quad \mathrm{P}(\mathit{succ} \mid \mathit{cyber}) = 1/2,$$
$$\mathrm{P}(\mathit{dalek}) = 2/5, \quad \mathrm{P}(\mathit{succ} \mid \mathit{dalek}) = 3/10$$

$$\begin{aligned}
& \mathrm{P}(\mathit{succ}) \\
= \; & \mathrm{P}(\mathit{cyber} \wedge \mathit{succ}) + \mathrm{P}(\mathit{dalek} \wedge \mathit{succ}) \\
= \; & \mathrm{P}(\mathit{cyber}) * \mathrm{P}(\mathit{succ} \mid \mathit{cyber}) + \mathrm{P}(\mathit{dalek}) * \mathrm{P}(\mathit{succ} \mid \mathit{dalek}) \\
= \; & 3/5 * 1/2 + 2/5 * 3/10 \; = \; 21/50
\end{aligned}$$

Example: Killer Robots

- cyberman and the dalek attack the Tardis daily.
- *cyber* has probability ½ of success.
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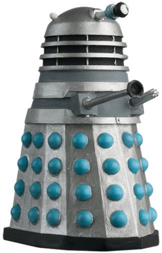

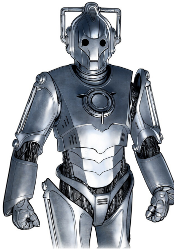

Example: Killer Robots

- cyberman and the dalek attack the Tardis daily.
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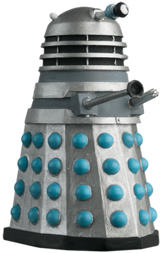

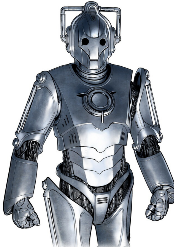

Example: Killer Robots

- cyberman and the dalek attack the Tardis daily.
- *cyber* has probability ½ of success.
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- Conditional probability: $\mathrm{P}(A \wedge B) = \mathrm{P}(A) * \mathrm{P}(B \mid A)$.

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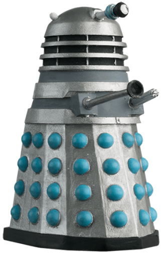

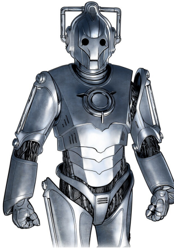

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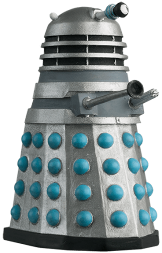

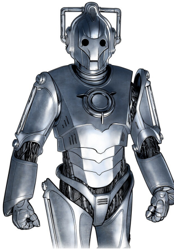

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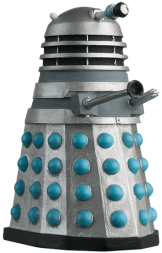
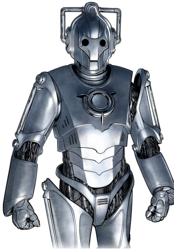
$$\mathrm{P}(\mathit{dalek}) = {}^2\!/_5, \quad \mathrm{P}(\mathit{succ} \mid \mathit{dalek}) = {}^3\!/_{10}$$

$$\begin{aligned} & \mathrm{P}(\mathit{succ}) \\ = {} & \mathrm{P}(\mathit{cyber} \wedge \mathit{succ}) + \mathrm{P}(\mathit{dalek} \wedge \mathit{succ}) \\ = {} & \mathrm{P}(\mathit{cyber}) * \mathrm{P}(\mathit{succ} \mid \mathit{cyber}) + \mathrm{P}(\mathit{dalek}) * \mathrm{P}(\mathit{succ} \mid \mathit{dalek}) \\ = {} & {}^3\!/_5 * {}^1\!/_2 + {}^2\!/_5 * {}^3\!/_{10} = {}^{21}\!/_{50} \end{aligned}$$





Computational Approach

$\mathrm{P}(\mathit{cyber}) = {}^{3}\!/_{5}, \mathrm{P}(\mathit{succ} \mid \mathit{cyber}) = {}^{1}\!/_{2},$

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$\mathrm{P}(\textit{cyber}) = {}^{3}\!/\!_{5}$, $\mathrm{P}(\textit{succ} \mid \textit{cyber}) = {}^{1}\!/\!_{2}$,
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Computational Approach

$\mathrm{P}(\mathit{cyber}) = \nicefrac{3}{5}, \mathrm{P}(\mathit{succ} \mid \mathit{cyber}) = \nicefrac{1}{2},$
$\mathrm{P}(\mathit{dalek}) = \nicefrac{2}{5}, \mathrm{P}(\mathit{succ} \mid \mathit{dalek}) = \nicefrac{3}{10}$

```
Tardis =
    if $\nicefrac{3}{5}$ then
        ( robot := cyber ;
          if $\nicefrac{1}{2}$ then attack := succ
          else attack := fail )
    else
        ( robot := dalek ;
          if $\nicefrac{3}{10}$ then attack := succ
          else attack := fail )
```

Computational Approach

$\mathrm{P}(\mathit{cyber}) = 3/5, \mathrm{P}(\mathit{succ} \mid \mathit{cyber}) = 1/2,$
$\mathrm{P}(\mathit{dalek}) = 2/5, \mathrm{P}(\mathit{succ} \mid \mathit{dalek}) = 3/10$

```
Tardis =
    if 3/5 then
        ( robot := cyber ;
          if 1/2 then attack := succ
          else attack := fail )
    else
        ( robot := dalek ;
          if 3/10 then attack := succ
          else attack := fail )
```

```
dtmc
const int cyber=1;
const int dalek=2;
const int succ=1;
const int fail=2;
module Tardis
   robot  : [1..2] init 1;
   attack : [1..2] init 1;
   s      : [0..3] init 0;
   [] s=0 -> 3/5: (robot'=cyber) & (s'=1)
           + 2/5: (robot'=dalek) & (s'=2);
   [] s=1 -> 1/2: (attack'=succ) & (s'=3)
           + 1/2: (attack'=fail) & (s'=3);
   [] s=2 -> 3/10:(attack'=succ) & (s'=3)
           + 7/10:(attack'=fail) & (s'=3);
   [] s=3 -> true;
endmodule
```

Computational Approach

$$\begin{aligned}
\mathit{Tardis} &= \textbf{if } \tfrac{3}{5} \textbf{ then } (\,\mathit{robot} := \mathrm{cyber}\ ;\ \textbf{if } \tfrac{1}{2} \textbf{ then } \mathit{attack} := \mathrm{succ} \textbf{ else } \mathit{attack} := \mathrm{fail}\,) \\
&\ \textbf{else } (\,\mathit{robot} := \mathrm{dalek}\ ;\ \textbf{if } \tfrac{3}{10} \textbf{ then } \mathit{attack} := \mathrm{succ} \textbf{ else } \mathit{attack} := \mathrm{fail}\,) \\
&= \tfrac{3}{10} * (\mathit{robot}, \mathit{attack} := \mathrm{cyber}, \mathrm{succ}) + \tfrac{3}{10} * (\mathit{robot}, \mathit{attack} := \mathrm{cyber}, \mathrm{fail}) \\
&\ + \tfrac{6}{50} * (\mathit{robot}, \mathit{attack} := \mathrm{dalek}, \mathrm{succ}) + \tfrac{14}{50} * (\mathit{robot}, \mathit{attack} := \mathrm{dalek}, \mathrm{fail})
\end{aligned}$$

- Probabilistic final states: assignments. Semantically equivalent to:
- Probability that $\mathit{attack}' = \mathrm{succ}$: $\tfrac{3}{10} + \tfrac{6}{50} = \tfrac{21}{50}$, the same answer as before.

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\textit{Tardis} &= \textbf{if } \tfrac{3}{5} \textbf{ then } (\, \textit{robot} := \text{cyber} \;;\; \textbf{if } \tfrac{1}{2} \textbf{ then } \textit{attack} := \text{succ} \textbf{ else } \textit{attack} := \text{fail} \,) \\
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&= \tfrac{3}{10} * (\textit{robot}, \textit{attack} := \text{cyber}, \text{succ}) + \tfrac{3}{10} * (\textit{robot}, \textit{attack} := \text{cyber}, \text{fail}) \\
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\mathit{Tardis} \;&=\; \textbf{if } {}^{3}\!/\!_{5} \textbf{ then } (\, \mathit{robot} := \text{cyber} \;;\; \textbf{if } {}^{1}\!/\!_{2} \textbf{ then } \mathit{attack} := \text{succ} \textbf{ else } \mathit{attack} := \text{fail} \,) \\
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&= {}^{3}\!/_{10} * (\textit{robot}, \textit{attack} := \text{cyber}, \text{succ}) + {}^{3}\!/_{10} * (\textit{robot}, \textit{attack} := \text{cyber}, \text{fail}) \\
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\end{aligned}$$

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```
module Tardis
  robot  : [1..2] init 1;
  attack : [1..2] init 1;
  t      : [0..1] init 0;
  [] t=0 -> 3/10:  (robot'=cyber) & (attack'=succ) & (t'=1)
            3/10:  (robot'=cyber) & (attack'=fail) & (t'=1)
            6/50:  (robot'=dalek) & (attack'=succ) & (t'=1)
            14/50: (robot'=dalek) & (attack'=fail) & (t'=1)
  [] t=3 -> true;
endmodule
```

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Outline

Introduction

Motivation

Prism’s Semantics

Probabilistic Predicative Programming

Conclusions

Further Work (1)

- Apply this semantics to unifying theories of uncertainty.
- Partially observable Markov decision processes, dynamic epistemic logic, . . .
- Research on describing and analysing uncertainty raises many questions.
- What would a unifying theory for uncertainty look like?
- What connects the semantics and tools that support different approaches?
- Can we establish more connections?
- Can we support probabilistic/statistical model checking with theorem proving?
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- Can we establish uncertainty properties using CbyC?

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Further Work (2)

- What about probabilistic refinement-based model checking?
- Can we qualify one target analysis tool for high assurance?
- What's the formal testing theory for a system with unknown MDP semantics?
- What are the testability hypotheses (in Gaudel's sense)?
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