Inhomogeneous Domain Walls

Existence and Propagation of Interfaces in Nanowires

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Domain Walls

Magnetization dynamics $\boldsymbol{m}(x,t) \in \mathbb{S}^2$ with spin-transfer governed by **Landau-Lifschitz-Gilbert-Slonczewski** equation

$$\partial_t \boldsymbol{m} - \alpha \boldsymbol{m} \times \partial_t \boldsymbol{m} = -\boldsymbol{m} \times \boldsymbol{h}_{\text{eff}} + \boldsymbol{m} \times (\boldsymbol{m} \times \boldsymbol{J}) .$$

For **axial symmetry** and **polarized spin-torque** in direction $\boldsymbol{e}_3 \in \mathbb{S}^2$:

$$\boldsymbol{h}_{\text{eff}} = \partial_x^2 \boldsymbol{m} + (h - \mu\, \boldsymbol{m}) \boldsymbol{e}_3 , \quad \boldsymbol{J} = \frac{\beta}{1 + c_{\text{cp}} m_3} \boldsymbol{e}_3 ,$$

damping $\alpha > 0$, applied field $h \in \mathbb{R}$, anisotropy $\mu \in \mathbb{R}$, and $\beta \geq 0$ as well as $c_{\text{cp}} \in (-1, 1)$ describe strength of spin-transfer and ratio of polarization.

Domain Walls (DWs) separate uniform (spatial) states, i.e. heteroclinics, and are relative equilibria (w.r.t. translation, rotation) of the form

$$\boldsymbol{m}(\xi, t) = \boldsymbol{m}_0(\xi) e^{i\varphi(\xi,t)} ,$$

$\xi = x - st$, $\varphi(\xi,t) = \phi(\xi - st) + \Omega t$, **speed** s, **frequency** Ω, and $\boldsymbol{m}_0(\xi) = (\sin\theta, 0, \cos\theta)^{\mathsf{T}}$. Define $q(\xi) := \phi' = (m_1 m_2' - m_2 m_1') \cdot (1 - m_3^2)^{-1}$.

Definition 1. *DW with constant ϕ, i.e. $q \equiv 0$, is called* **homogeneous** *and* **inhomogeneous** *otherwise. It is called* **flat** *if q has a limit at $|\xi| \to \infty$ and* **non-flat** *otherwise.*

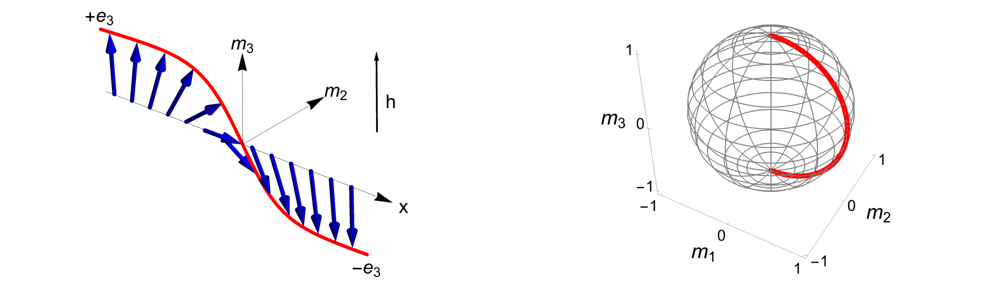


Figure 1. Homogeneous DW profile (left) and projection onto sphere (right). Parameters: $\alpha = 0.5, \beta = 0.1, \mu = -1, h = 50, c_{\text{cp}} = 0, s_0 = 19.92, \Omega_0 = 40.4$.

Existence

If $c_{\text{cp}} = 0$ (or $\beta = 0$), family of explicit homogeneous DWs for $\mu < 0$ (nanowire) known[1,2]

$$\boldsymbol{m}_0 = \left(\operatorname{sech}\left(\sigma\sqrt{-\mu}\xi\right), 0, -\tanh\left(\sigma\sqrt{-\mu}\xi\right)\right)^{\mathsf{T}},$$

parameteried by $\Omega_0 = \frac{h + \alpha\beta}{1+\alpha^2}$, $s_0^2 = -\frac{(\beta - \alpha h)^2}{\mu(1+\alpha^2)^2} > 0$ ($\sigma = 1$ for $s_0 > 0$, $\sigma = -1$ for $s_0 < 0$). Smooth extension to $s_0 = 0$ (standing front) as $h \to \beta/\alpha$ (both sides).

Question: what happens for $c_{\text{cp}} \neq 0$? Idea, perturb away from $\boldsymbol{m}_0$! Focus on right-moving DWs, i.e. $s \geq 0$.

Theorem 1 (point-to-point). *For any parameter set (α, β, h, μ) in case $\beta/\alpha \leq h < h^*$, $h^* := \beta/\alpha - 2\mu - 2\mu/\alpha^2$, or $h > h^*$ with $\mu < 0$, $\boldsymbol{m}_0$ lies in a smooth family $\boldsymbol{m}_{c_{\text{cp}}}$ of DWs, where in the first case these are locally unique near $\boldsymbol{m}_0$ and (s, Ω) are functions of parameters close to (s_0, Ω_0). Moreover, if $c_{\text{cp}} = 0$ and $(s, \Omega) \neq (s_0, \Omega_0)$, or $c_{\text{cp}} \neq 0$, these are* **inhomogeneous flat** *DWs.*[3]

Theorem 2 (point-to-cycle). *In case $h = h^*$ with $\mu < 0$, consider $\boldsymbol{m}_0$ for parameters satisfying $\Omega = s^2/2 + \beta/\alpha$ and varying (c_{cp}, s, h). Flat DWs occur at most on a surface in the s, h, c_{cp} parameter space and, for $\beta \neq 0$, satisfy $|s_\varepsilon|^2 + |h_\varepsilon|^2 = \mathcal{O}(|c_{\text{cp}}|^3)$, more precisely (1). Otherwise DWs are* **non-flat**, *in particular all DWs* **not equal** *to $\boldsymbol{m}_0$ for $c_{\text{cp}} = 0$ or $\beta = 0$ are* **non-flat**.[3]

Remark 1: For $h = h^*$, existence of 'Energy' at target state $\theta = \pi$, i.e. $m_3 = -1$ $\Rightarrow$ right asymptotic limit ($\xi \to \infty$) of perturbed (in c_{cp}) heteroclinic is either perturbed equilibrium or periodic orbit. Hence variation does not have a limit in general, **but** energy does and difference between perturbed equilibrium and heteroclinic at $\theta = \pi$, w.r.t. energy:

$$(1) \quad \frac{(1+\alpha^2)\pi^2}{\alpha\rho^2\mu\sqrt{-\mu}} \left(-\frac{\mu(1+\alpha^2)(4+\alpha^2)}{\alpha} \overline{s}^2 - \frac{2\sqrt{-\mu}(2+\alpha^2)}{\alpha} \overline{s}\overline{h} + \overline{h}^2 \right) + \text{h.o.t.},$$

where $\overline{s} = s - s_0, \overline{h} = h - h_0, \rho = \exp(\pi/\alpha) - \exp(-\pi/\alpha)$.

Remark 2: Existence results also valid for left-moving walls, hence $\forall\, h \in \mathbb{R}$. Moreover, also in case $\beta = 0$, i.e., for the (well-known) **LLG** equation.

Numerical Continuation

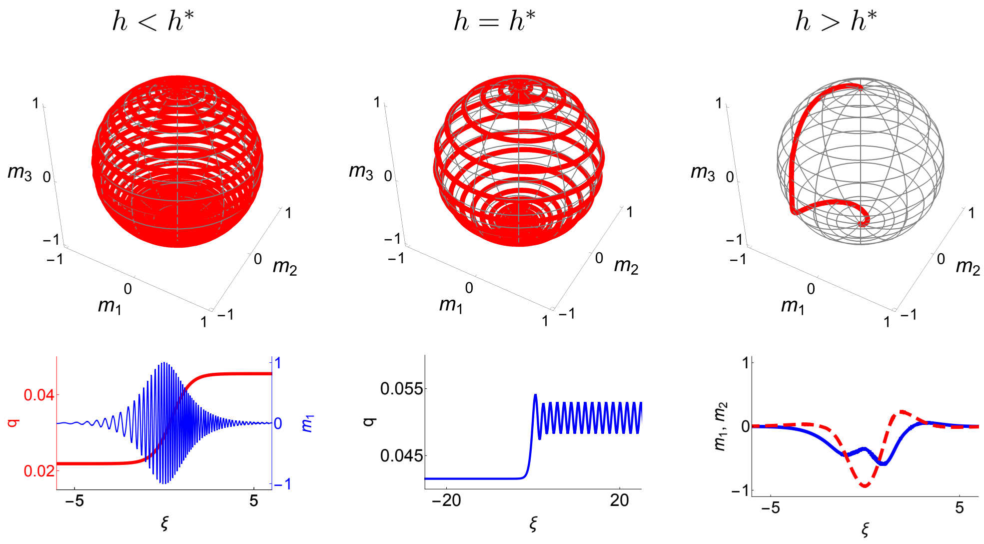


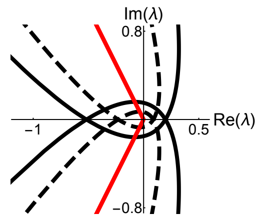
Figure 2. DWs on $\mathbb{S}^2$ (uppper panel), computed by continuation up to $c_{\text{cp}} = 0.5$ for $\alpha = 0.5$, $\beta = 0.1$, $\mu = -1$ fixed. Left column: $h = 0.5$, $s = 0.112$, $\Omega = 0.447$ and zoom-in on m_1 (blue) and q (red) component (lower left). Note change of frequency of m_1. Center column (**point-to-cycle**): $h = h^* = 10.2$, $s = 4$, $\Omega = 8.2$ and zoom-in on q component (lower center). Right column: $h = 50$, $s = 19.92$, $\Omega = 40.4$ and zoom-in on m_1 (blue solid) and m_2 (red dashed) component (lower right).

Propagation

Perturbations tangential to sphere, asymptotic state $\boldsymbol{e}_3$ ($\xi \to -\infty$) for $s \geq 0$ is L^2-stable for $h > \beta^+/\alpha + \mu$, while $-\boldsymbol{e}_3$ ($\xi \to +\infty$) stable for $h < \beta^-/\alpha - \mu$ and unstable for $h > \beta^-/\alpha - \mu$ with $\beta^\pm := \beta/(1 \pm c_{\text{cp}})$ (Hopf-type instability with frequency $\beta^-/\alpha)^2$. Assume $\frac{c_{\text{cp}}\beta/\alpha}{1 - c_{\text{cp}}^2} > \mu$.

Due to explicitly known absolute spectrum of fronts (pulled invasion into $-\boldsymbol{e}_3$), (linear) **spreading speed** for $\beta^-/\alpha - \mu \leq h$ is

$$s^* = 2\sqrt{\frac{h + \mu - \beta^-/\alpha}{1+\alpha^2}} .$$



Spectra of $-\boldsymbol{e}_3$: essential (solid black), weighted ($\eta = -\alpha s^*/6$) essential (dashed black), absolute (solid red). $\alpha = 0.5, \beta = 0.1, \mu = -1, c_{\text{cp}} = 0, h = 2, s = s^* = 1.6, \Omega = (s^*)^2/2 + \beta^-/\alpha = 1.48$.

Remark 3: Selection of (linear) **spreading frequency** numerically observed to be $\Omega^* = (s^*)^2/2 + \beta^-/\alpha$, hence selection of **non-flat** DWs (see **Theorem 2**). Note that frequency does not effect real part of spectra.

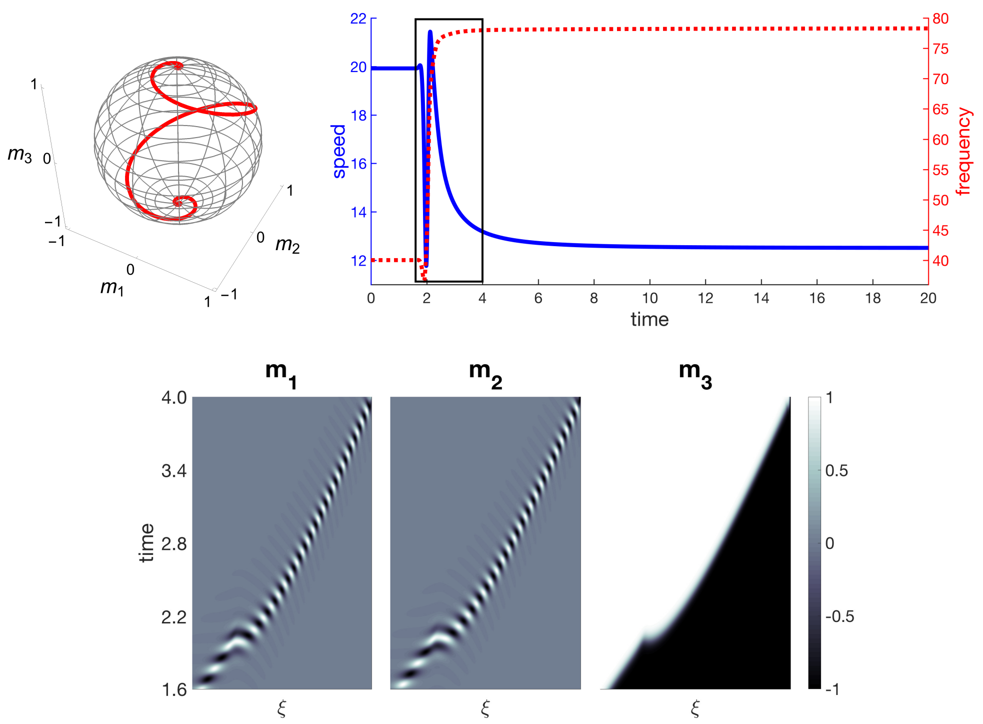


Figure 3. Selected inhomogeneous DW of full PDE dynamics over time projected onto $\mathbb{S}^2$ (upper left) with **freezing** of speed and frequency (upper right). Perturbed homogeneous DW as initial data with $\alpha = 0.5, \beta = 0.1, \mu = -1, h = 50, c_{\text{cp}} = 0$. Selected spreading speed $s^* = 12.5$ and frequency $\Omega^* = 78.28$. Snapshot of non-freezed components (lower row) in transition (from $t = 1.6$ to $t = 4$, cf. black box upper right).

[1] Goussev, A., & Robbins, J., & Slastikov, V. (2010). Domain-Wall motion in ferromagnetic nanowires driven by arbitrary time-dependent fields: An exact result. In Physical review letters, 104(14):147202

[2] Melcher, C., & Rademacher, J. (2017). Pattern formation in axially symmetric Landau-Lifshitz-Gilbert-Slonczewski equations. In Journal of Nonlinear Science 27.5: 1551-1587

[3] Siemer, L., & Ovsyannikov, I., & Rademacher, J. (Preprint). Inhomogeneous domain walls in spintronic nanowires.

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