Stabilization by transport noise and enhanced dissipation in the Kraichnan model

Seminar talk in Applied Analysis Seminar on Nov. 23, 2021

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Thanks to the work of Arnold, Crauel, and Wihstutz it is known that for any self-adjoint operator T acting on a finite dimensional space with the negative trace the corresponding linear equation $dx_t = Tx_t dt$ can be stabilized by a noise, i.e. there exists operator-valued Brownian motion W such that the solution of $dx_t + dWx_t = Tx_t dt$ vanishes a.s. for any initial value $x_0 = x$. The goal of the talk is to extend this theorem to infinite dimensions. Namely, we prove that the equation $du_t = (\Delta + C)\, u_t dt$ can be noise stabilized and that an arbitrary large exponential rate of decay can be reached. The sufficient conditions on the noise are shown to be satisfied by the so-called Kraichnan model for stochastic transport of passive scalars in turbulent fluids.