Non-autonomous semilinear parabolic equations and Schaefer's fixed point theorem

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We show the existence of solutions of nonlinear non-autonomous Cauchy problems

$$\partial_t u(t,x) - \nabla_x \cdot (a(t,x)\nabla_x u(t,x)) = f(t,x,u(t,x),\nabla u(t,x)), \qquad u(0,\cdot) = u_0$$

for a bounded open set $\Omega \subseteq \mathbb{R}^n$. The coefficient matrix a is supposed to be symmetric, uniformly elliptic, Lipschitz continuous w.r.t. $t \in (0,\tau)$ and measurable w.r.t. $x \in \Omega$; the nonlinearity f is required to be continuous and to satisfy a suitable growth condition. We show that, given $u_0 \in H_0^1(\Omega)$, there exists $u \in L_2(0,\tau;H_0^1(\Omega)) \cap H^1(0,\tau;L_2(\Omega))$ solving the problem mentioned above.

The proof relies on Schaefer's fixed point theorem. In the course of the proof one uses maximal regularity properties of solutions of inhomogeneous linear problems and compact embeddings of vector-valued Sobolev spaces.

The result is inspired by [ArCh10].

The talk is based on joint work with Wolfgang Arendt and Jürgen Voigt.

[ArCh10] W. Arendt, R. Chill: *Global existence for quasilinear diffusion equations in isotropic nondivergence form.* Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **IX**, 523–539 (2010).