Energy Minimization in DAG Scheduling on MPSoCs at Run-Time: Theory and Practice

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Introduction

DVFS (Dynamic Voltage and Frequency Scaling)

- Objective: decrease the energy consumption of processors
- Constraint: respect a strong deadline

Motivation example

- Application run multiple times, exact characteristics depend on the workload
- Some settings [Choudhury et al. 2007, Singh et al. 2013] : compute a (pessimistic) pseudo-schedule offline, adapt it online
- Ideal: compute online (i.e., fast) a solution with low energy consumption

Problematic

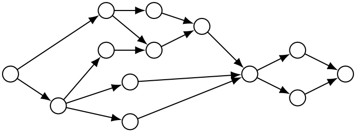
Are there theoretically-guaranteed algorithms that are fast enough to be executed at run-time?

Preliminaries

General statement of the problem

- DAG $G = (V, E)$ of n tasks of known lengths w_j
- m cores, whose speeds can be modified between two tasks
- Strong deadline D

$\Rightarrow$ Minimize the energy



Energy model: DVFS

$$Power = speed^{\alpha}, \text{ for some } \alpha > 1$$

Energy consumed for a task of length w_j, run at speed s_j

- Execution time: $x_j = w_j / s_j$
- Energy $E_j = x_j \cdot s_j^{\alpha} = w_j \cdot s_j^{\alpha-1} = w_j^{\alpha} / x_j^{\alpha-1}$

$\Rightarrow$ Objective: minimize $\sum E_j$

Four variants of the problem

Two scenarios

- SPEED&SCHEDULING – the problem is to:
  - decide at which speed each task is run;
  - schedule each task to a core, respecting precedences.
- SPEEDSCALING – the task-to-core mapping and each core's execution order is fixed
  The problem is to:
  - decide the speed of each task

Two speed models

- Continuous speeds: all speeds in $[s_{min}, s_{max}]$
- Discrete speeds: choose speeds among $v_1, \ldots, v_k$

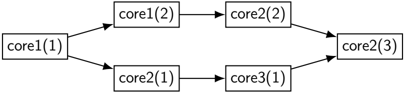
Theoretical guarantees targeted: e.g., 2-approximation

- Deadline is always met (if feasible)
- Energy consumed is at most 2 times the best possible

Outline

Optimal polynomial solution: convex program
[Aupy et al. 2013]

Note: include each core's order into the precedence constraints
$\longrightarrow$ execution time = critical path



Convex program computing the optimal speeds

x_j: execution duration of task j $(x_j = w_j/s_j)$
d_j: completion time of task j

$$\min \sum_{j \in V} \frac{w_j^\alpha}{x_j^{\alpha-1}}$$

$$\begin{aligned} \text{s.t.} \quad d_j &\leq D, && \forall j \in V \\ x_j &\leq d_j, && \forall j \in V \\ d_j + x_k &\leq d_k, && \forall (j,k) \in E \\ w_j/s_{max} \leq x_j &\leq w_j/s_{min}, && \forall j \in V. \end{aligned}$$

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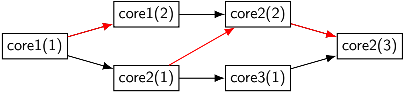

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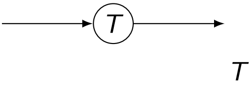
Optimal linear-time solution for Series-Parallel graphs

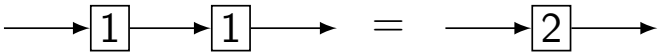
[Prasanna Musicus 1996] in an other context

Series-Parallel graph



Main algorithm idea: each subgraph is equivalent to a single task

- $w(T) = w_T$
- $w(G_1 ; G_2) = w(G_1) + w(G_2)$:



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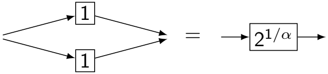

Algorithm sketch (drawback: ignores s_{min} and s_{max})

1. Compute the equivalent task of G: assign it the speed $w(G)/D$
2. Propagate the speed assignment through the graph structure (series: conserve speed, parallel: conserve execution time)

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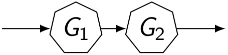

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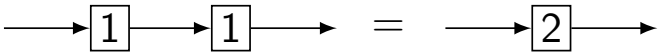

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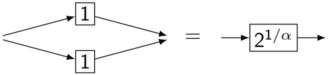

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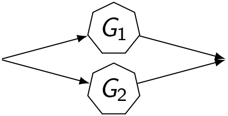

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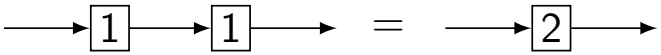

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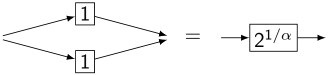

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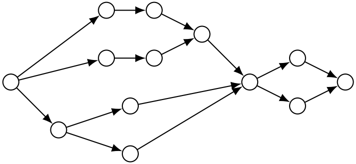

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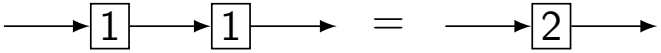

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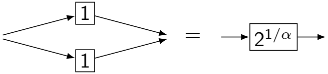

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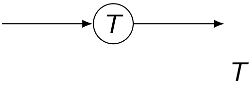

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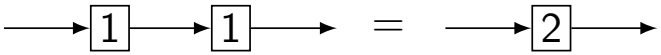

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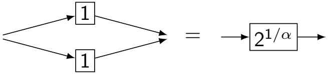

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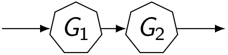

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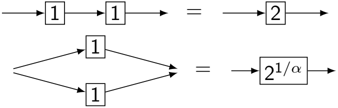

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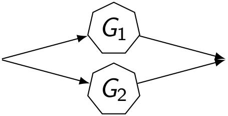

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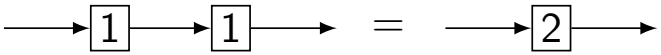

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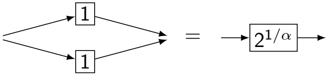

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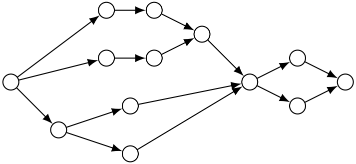

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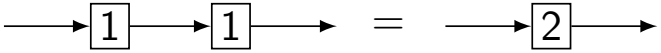

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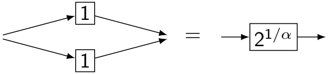

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Outline

Approximate solution

similar to [Bampis, Letsios, Lucarelli 2014]

Issue to obtain an approximation-algorithm

- with fixed speeds, scheduling is NP-hard
- need to assume that the deadline is loose to be able to meet it

Theorem

If OPT uses speeds at most $s_{max}/2$*, there is a* $2^{\alpha-1}$*-approximation.*

Algorithm sketch

- Solve the previous CP adding the constraints ($m = \#$cores):

$$\sum_{j \in V} \boldsymbol{x_j}/m \leq D/2 \quad ; \qquad \boldsymbol{d_j} \leq D/2 \qquad \forall j \in V$$

- Use Graham's list-scheduling or any such simple algorithm
- If there is some slack towards the deadline, scale down the speeds

Outline

Recall

Results for continuous speeds

	Exact Solution	Approximate Solution
SpeedScaling	Convex Program	SP-graphs (restricted instances)
Speed&Scheduling		Convex Program + Rounding + List Scheduling ($2^{\alpha-1}$-approx)

This section: discrete speeds

- $v_1 \leq v_2 \leq \cdots \leq v_k$
- Define $r := \max_i \frac{v_{i+1}}{v_i}$

Outline

Optimal exponential-time solution: ILP

$\boldsymbol{y_{i,\ell}}$: boolean variable deciding whether task i is run at speed v_ℓ
$\boldsymbol{d_i}$: completion time of task i

$$\text{minimize } \sum_{i \in V} w_i \left(\sum_{\ell \leq k} v_\ell^{\alpha-1} \boldsymbol{y_{i,\ell}} \right)$$

$$\boldsymbol{d_i} \leq D \qquad \forall i \in V$$

$$\left(\sum_{\ell \leq k} \frac{w_i}{v_\ell} \boldsymbol{y_{i,\ell}} \right) \leq \boldsymbol{d_i} \qquad \forall i \in V$$

$$\boldsymbol{d_i} + \left(\sum_{\ell \leq k} \frac{w_j}{v_\ell} \boldsymbol{y_{j,\ell}} \right) \leq \boldsymbol{d_j} \qquad \forall (i,j) \in E$$

$$\sum_{\ell \leq k} \boldsymbol{y_{i,\ell}} = 1 \qquad \forall i \in V$$

$$\boldsymbol{y_{i,\ell}} \in \{0,1\} \qquad \forall i \in V, \forall \ell \leq k.$$

Approximate solution

Simple algorithm

1. Compute the optimal continuous-speed solution (with $s_{min} = v_1, s_{max} = v_k$)
2. Round up each speed

Note: we can use the fast SP-graph algorithm or the convex program

Theorem

This algorithm is a $r^{\alpha-1}$-approximation.

Recall: $r = \max_i \frac{v_{i+1}}{v_i}$

Outline

The SPEED&SCHEDULING setting

Optimal exponential-time solution via an ILP

- Needs $n(n+m)$ boolean variables: really prohibitive complexity

Approximate solution

- Combine both previous approximation schemes (assuming OPT uses speeds at most $v_k/2$)
- Compute the approximate continuous speed solution then round up the speeds
- Guarantee: $(2r)^{\alpha-1}$-approximation

Outline

The SpeedScaling setting

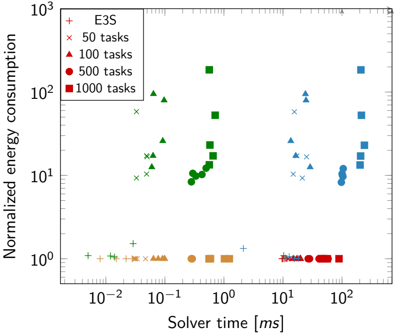
	Approximate Solution	Exact Solution
Continuous speeds	SP-graph (exact solution)	Convex Program
Discrete speeds	SP-G + rounding ($r^{\alpha-1}$-approx)	ILP

Datasets (all SP-graphs)

- E3S ($\approx$ 10 tasks)
- GENOME (Pegasus, 50 to 1000 tasks)
- Discrete speeds: 20 equally distributed

Results (bottom left is better)

- SP-G: 1ms for 1000 tasks
- CVX: 15ms for 100 tasks
- Discrete speeds: almost optimal



The SPEED&SCHEDULING setting

	Approximate Solution	Exact Solution
Continuous speeds	Convex Program + List Scheduling ($2^{\alpha-1}$-approx)	
Discrete speeds	Convex Program + Rounding + List Scheduling ($(2r)^{\alpha-1}$-approx)	Prohibitive ILP

Convex Program running time:

- 100 tasks in 25ms
- 500 tasks in 75ms
- 1000 tasks in 140ms

Conclusion

Results

- SpeedScaling, SP-graphs: almost-optimal solution can be computed very fast
- Other settings: guaranteed algorithms exist but are slower benefits depend on the application

Future work

- Integration of such algorithms in a run-time resource management framework