PhD project R1-5

Inverse Problems and Deep Learning (1st cohorte)

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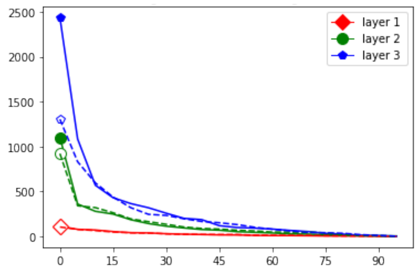
RTG 2224

Singular Values for ReLU Layers

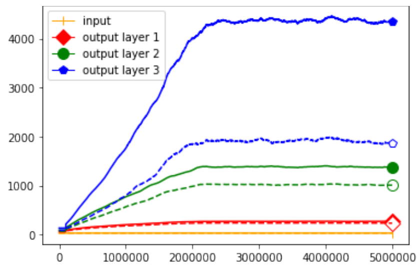
- We generalize singular values to ReLU layers and used them and the Gaussian mean width to analyze the behavior of data inside neural networks.
- Definition of (data independent) ReLU singular values:

$$s_k = \min_{\operatorname{rank} L \leq k} \max_{x \in \mathcal{B}} ||\operatorname{ReLU}(Ax) - \operatorname{ReLU}(Lx)||_2. \quad (1)$$

Data dependend ReLU singular values of a network. Misclassified (dashed) vs. correctly classified (solid) data.



The Gaussian mean width of the data after dfferent layers over the course of the training. Again misclassified vs. correctly classided data.



Published: IEEE Transactions on Neural Networks and Learning Systems

Regularization by architecture

- We analyze the idea of the Deep image prior (DIP) in terms of a variational formulation and the implicit function theorem to find closed form solutions for special cases.
- The DIP idea: Solve an inverse problem via

$$\min_{\Theta} \|A\varphi_B(z) - y^\delta\|^2, \quad (2)$$

where $\varphi_B(z)$ is an untrained neural net with paramters B and a fixed random input z.

We analyze infinitly deep recurrent networks consiting of layers of the form

$$x^{k+1} = \operatorname{prox}_{\lambda\alpha R}\left(x^k - \lambda B^*(Bx^k - y^\delta)\right). \quad (3)$$

For these nets the solution can be rewriten via the implicit function theorem and efficiently analyzed. E.g., for the case $R(x) = \frac{1}{2}\|x\|^2$, one finds that the problem simply reduces to

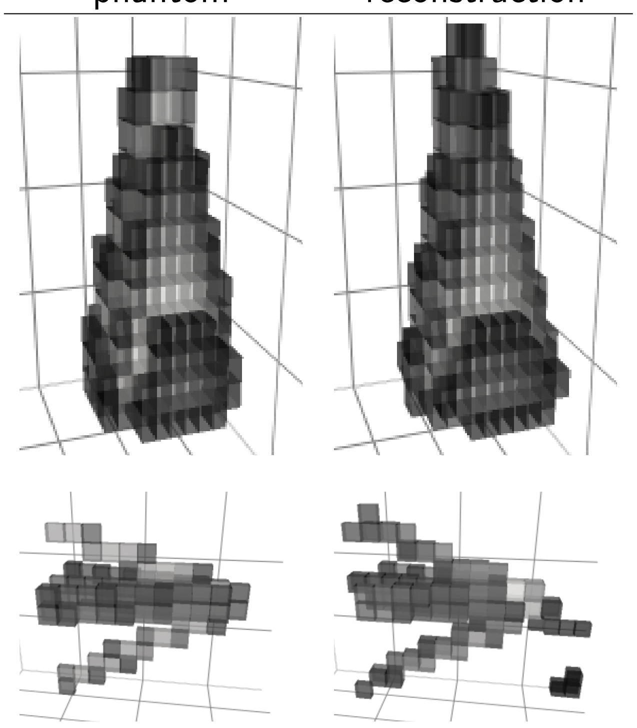
$$\min_B \frac{1}{2}\|A(B^*B + \alpha I)^{-1}B^*y^\delta - y^\delta\|^2. \quad (4)$$

Published: Springer, Journal of Mathematical Imaging and Vision

Deep image prior for 3D magnetic particle imaging

- We created a DIP (see explation in top right of the poster) with a three-dimensional architecture such that we could apply it to the preclinical imaging modality "magnetic particle imaging" (MPI)
- We created a large scale comparison between it and classical reconstruction methods used for MPI based on the `Open MPI dataset`.
- We found that overall our DIP approch outperformed all classical methods in terms of PSNR and SSIM.

phantom	reconstruction



Ground Truth Free Denoising by Optimal Transport

- The idea is to denoise data based on noise samples (from a distribution p_η) and noisy samples (from a distribution p_{y^δ}) only.
- We only assume that the noise is independet of the data and additive.
- We start out by assuming that a perfect denoiser G^* has the properties:

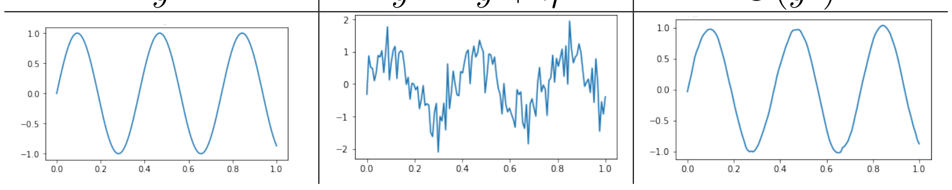
$$(G^*_\# p_{y^\delta}) * p_\eta = p_{y^\delta}, \quad (5)$$

and

$$(\mathrm{id} - G^*)_\# p_{y^\delta} = p_\eta. \quad (6)$$

- We train a network G to apprximate a solution to these equations in terms of the Wasserstein metric. We do so by formulating the problems as a novel Wasserstein Generative Adversarial Network setting.
- We find for different scenarios (including images) that we are on par or better than state-of-the-art unsupervised denoisers like BM3D.

y	$y^\delta = y + \eta$	$G(y^\delta)$



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