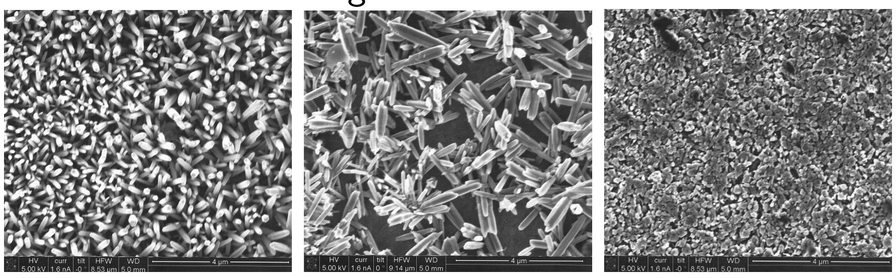
Electromagnetic inverse scattering problem

Alexander Konschin

RTG 2224

Scattering on nano-structured surfaces

- Non-destructive testing method for nano structures.



- Electromagnetic wave propagation modeled by Maxwell's equations

$$\nabla \times (\mu^{-1}\nabla \times E) - \omega^2 \left(\varepsilon + \mathrm{i}\frac{\sigma}{\omega}\right) E = F \quad \text{in } \mathbb{R}^3,$$

where permeability μ is assumed to be periodic and permittivity ε is also periodic but locally perturbed.

- Electromagnetic wave propagation in TE mode modeled by Helmholtz equation

$$\Delta u + k^2 n^2 u = f \quad \text{in } \mathbb{R}^3,$$

where refractive index n^2 is assumed to be periodic but locally perturbed.

- Main tool: Bloch-Floquet transform defined by

$$(\mathcal{J}\phi)(\alpha, x_1, x_2, x_3) = \sum_{j \in \mathbb{Z}^2} \phi(x_1 + j_1, x_2 + j_2, x_3)\, e^{\mathrm{i}\alpha\cdot j}.$$

- Theorem: Under some assumptions there exists a unique solution to the Maxwell's equations and the Helmholtz equation. [1] [2]

Discretization

- Bloch-Floquet transform gives a family of quasi-periodic scattering problems with coupling.
  ⇒ suits perfectly for discretization and parallelizes greatly
- Finite-element space: locally constant functions in α, and Nédéc or Lagrange elements in space. Solve large linear equation system by GMRES combined with incomplete LU decomposition. [2] [3]

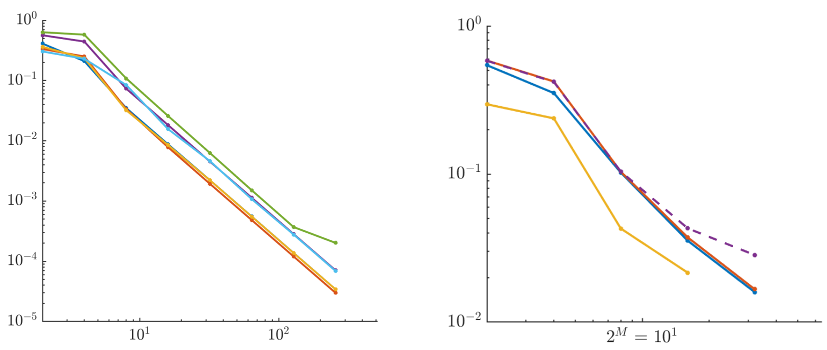
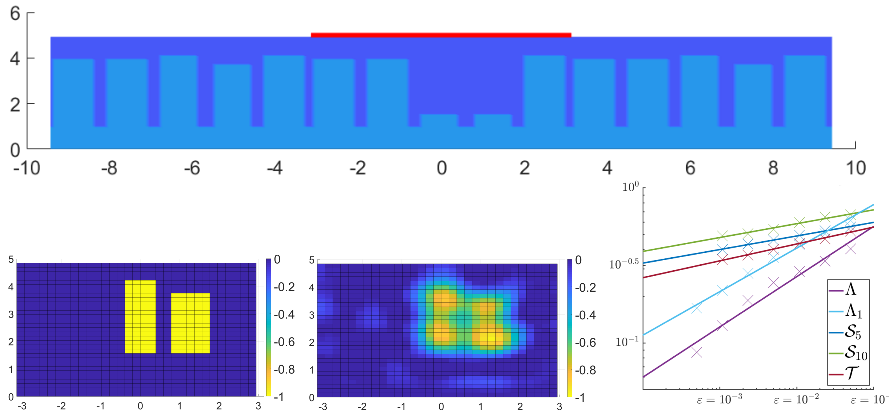


Figure 1: L^2-error related to discretization in space (Helmholtz and Maxwell).

Inverse Problem

- Goal: detect perturbation in periodic structure having measurements of the scattered wave.
- Measurement operators: Λ measures full wave, $\mathcal{S}$ measures near-field in one period and $\mathcal{T}$ measure far field of scattered wave.
- Theorem: Under some assumptions the measurement operators are injective, ill-posed and Fréchet differentible.
- Newton method gives nice results: [2]



- Reconstruction also works in 3D: [2] [3]

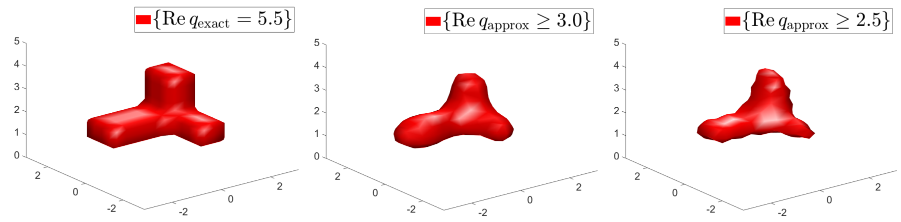


Figure 2: Middle: Helmholtz problem, right: Maxwell problem.

Factorization method

- The Factorization method is a fast imaging method for recontructing the support of the perturbation.
- Let F be the far field operator, $F_\# := |(\mathrm{Re}\, F)| + (\mathrm{Im}\, F)$.
- Theorem: Under some assumptions the operator $F_\#$ is strictly positive and

$$z \in \text{ support of perturbation} \quad \Leftrightarrow \sum_{j=1}^{\infty} \frac{|(\phi_z^\infty, \psi_j)_{L^2(S)}|^2}{\lambda_j} < \infty,$$

where ϕ_z^∞ is the far field of fundamental solution and $\{\lambda_j, \psi_j\}_{j=1}^\infty$ the eigen system of $F_\#$. [4]

- Numerical results clearly show the perturbed part:

[1] Konschin, A. (2019, August). Electromagnetic wave scattering from locally perturbed periodic inhomogeneous layers. In: submitted
[2] Konschin, A. and Lechleiter, A. (2019, November). Reconstruction of a local perturbation in inhomogeneous periodic layers from partial near-field measurements. In: Inverse Problems 35.11, S. 114006
[3] Konschin, A. (2019, September). Numerical scheme for electromagnetic scattering on perturbed periodic inhomogeneous media and reconstruction of the perturbation. In: submitted
[4] Haddar, H. and Konschin, A. (2019, March). Factorization Method for Localization of a Local Perturbation in Inhomogeneous Periodic Layers from Far Field Measurements. In: Inverse Problems and Imaging

DFG
Universität Bremen