PhD projects R2 – 1st cohort

R2-4: Online Optimal Parameter Identification in Dynamic Controlled Environments
R2-1: Towards Finding Preferable Minima in Nonl. Progr. by Decomposition Techn.

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RTG 2224

Online Parameter Identification

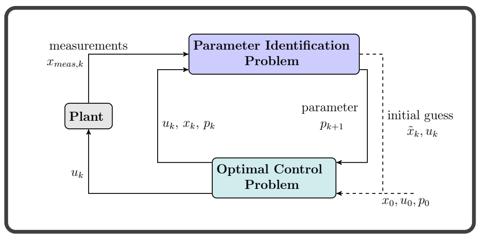
Model identification of **complex nonlinear systems** is difficult and can result in

- **Models depending on unknown or time-varying parameters,**
- Unexpected system behavior,
- Deviations in predicted costs.

The aim of the PhD project is to **provide online parameter updates** to stay closer to offline computed optimal trajectories or predicted costs. An enlargement of the controller's area of convergence may be achieved.

Combine Parameter Identification and Control Methods

- PI + Real-time adaptive LQR controller
- Repeated PI + nonlinear OC (Sequential loop)[1]
- PI + nonlinear model predictive control (MPC)



Parameter identif. (PI)

Find states and parameters for measurements $x_{meas,i} \in \mathbb{R}^n$ so that:

$$\min_{x,p} \frac{1}{N}\sum_{i=1}^{N} \|x(t_i,p) - x_{meas,i}\|^2,$$
$$\text{s.t. } \dot{x}(t,p) = F(x(t),u(t),p).$$

Optimal control (OC)

Find controls $u(t)$ that optimize control effort and distance to final point:

$$\min_{x,u} \int_0^T \ell(t,x(t),u(t))dt + \Phi(x(0),x(T))$$
$$\text{s.t. } \dot{x}(t,p) = F(x(t),u(t),p),$$
$$\varphi(x_0,\dot{x}_0) = 0.$$

Real-time adapted Riccati controller

Solve parameter dependent LQR problem as optimization problem with feedback control law $u(t) = -K(p)x(t)$

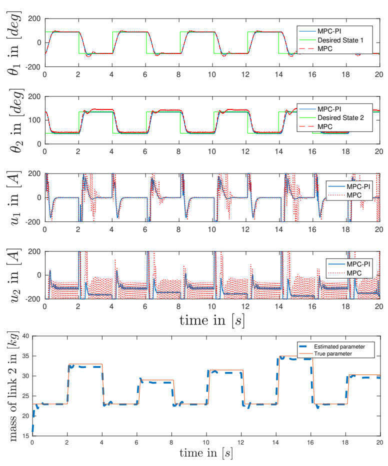
$$\min_{K(p)} \int_0^\infty x^T Q(p)x + x^T K(p)^T R(p)K(p)x dt$$
$$\text{s.t. } \dot{x}(t) = A(p)x(t) - B(p)Kx(t),$$
$$x(0) = x_0(p).$$

and update solution $K(p_0)$ with offline computed parametric sensitivities $\frac{dK(p)}{dp}(p_0)$

$$\tilde{K}(p) = K(p_0) + \frac{dK(p)}{dp}(p_0) \triangle p.$$

Applications

- **Benchmark example**: inverted pendulum on a cart.
- Nonlinear adaptive MPC for an **idealized robot manipulator** with time-varying payloads .[2]
- Examination of parameter identification methods: **Real-world application** on industrial robot. [3]



States, controls and mass parameter for idealized robot manipulator with adaptive nonlinear MPC.

Decomposition in Nonlinear Programming

Consider problems with **intricate sub-systems** ψ, implicitly given via $G(x,p) = 0$. Equivalent NLP formulations:

$$\min_p \quad F(\psi(p),p) \qquad \text{(Reduced)}$$
$$\min_{x,p} \quad F(x,p) \quad \text{s.t. } G(x,p) = 0 \qquad \text{(Decomposed)}$$

Decomposition offers several advantages, e.g.

- Robustness, Stability
- Beneficial algorithmic behavior

WORHP We Optimize Really Huge Problems

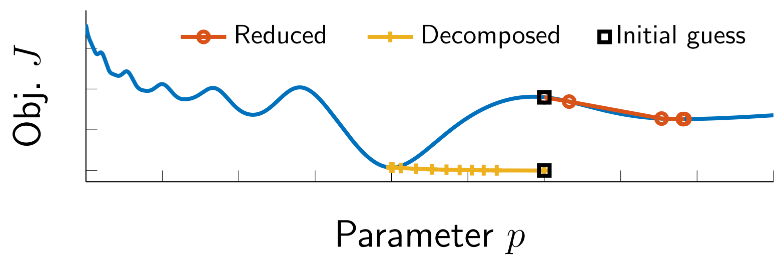
Following blocks: Examples for ψ

Parameter Identification: ψ is ODE solution

Fit a dynamic model to given data by optimizing parameters:

$$\min_{p,\,x_a} \quad J = \frac{1}{2}\sum_{i=1}^{\bar{N}} \frac{1}{\sigma_i^2} \left\| x|_{\mathcal{J}}(\bar{t}_i; x_a, p) - \bar{x}^{[\bar{t}_i]} \right\|_2^2$$
$$\text{s.t.} \quad \dot{x}(t) = f(t, x(t), p), \quad t \in [t_a, t_b], \quad x(t_a) = x_a$$

Algorithmic behavior of WORHP for a simple pendulum



Numerical comparisons of single/multiple shooting and full discretization w.r.t. finding **desired solutions**.

- Applications to **robotic system**[3,4]
- Development of **homotopy-optimization approach**[5]

Bilevel Programming: ψ is NLP solution

NLP with nested, parameterized lower-level problem (LL):

$$\min_{x,y} \quad F(x,y)$$
$$\text{s.t.} \quad G(x,y) \le 0 \text{ with } y \in \arg\min_s \{f(x,s)\colon g(x,s) \le 0\}$$

Embed process of numerically solving LL into NLP[6]:

$$\min_{x,\,y_k,\,\lambda_k} \quad F_N$$
$$\text{s.t.} \quad \nabla^2 \ell_k(y_{k+1} - y_k) + \nabla f_k + \nabla g_k^{\mathsf{T}} \lambda_{k+1} = 0, \quad \lambda_k \ge 0,$$
$$g_k + \nabla g_k(y_{k+1} - y_k) \le 0, \quad G_N \le 0$$

- Original solutions can be **recovered**
- Compete against KKT approach on **test library**
- Initialization & **adaptivity** strategies

[1]R., Flaßkamp, & Büskens. (2018), Sequential solution of parameter identification and optimal control problems for robotic systems. Proc. Appl. Math. Mech., 18: e201800099.
[2]R., Flaßkamp, & Büskens. (Forthcoming), Model Predictive Control with Online Nonlinear Parameter Identification for a Robotic System. Proc. Intern. Conf. (CoDIT 2020).
[3]Sch., R., Flaßkamp, & Büskens. Parameter Identification for Dynamical Systems Using Optimal Control Techniques, 2018 European Control Conference (ECC), Limassol, 2018, pp. 137-142.
[4]Sch., Flaßkamp, & Büskens. (2018), A Numerical Study of the Robustness of Transcription Methods for Parameter Identification Problems. Proc. Appl. Math. Mech., 18: e201800101.
[5]Sch., Flaßkamp, Fliege, & Büskens. (2019), A Combined Homotopy-Optimization Approach to Parameter Identification for Dynamical Systems. Proc. Appl. Math. Mech., 19: e201900266.
[6]Sch., Fliege, Flaßkamp, & Büskens. A Reformulation Technique for Nonlinear Bilevel Programs Based on Sequential Quadratic Programming. (in preparation)

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