PhD projects R3-10, R1-6

Learning Tikhonov Functionals for Inverse Problems (2nd cohorte)

Deep Image Prior Approaches to Inverse Problems (2nd cohorte)

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RTG 2224

Learning Tikhonov Functionals

We consider classical inverse problems given by a linear or non-linear operator $A : X \to Y$, which maps $x \in X$ to $y \in Y$. The related inverse problem asks to determine an approximation to $x^\dagger$ from given noisy measurements

$$y^\delta = Ax^\dagger + \eta.$$

This can be achieved by minimizing Tikhonov functionals of type

$$\mathcal{L}(x) = \|Ax - y^\delta\|^2 + \alpha R(x),$$

where $R : X \to \mathbb{R}$ is called regularization term and $\alpha \in \mathbb{R}^+$ is a scalar weighting factor.

- Traditional approaches use handcrafted regularizers to introduce certain properties to the solution
- In many cases much more advanced methods are needed on real data
- Learn a parameterized version R_θ directly from data[1,2]
- Investigate the choice and influence of different network architectures on the regularizing properties
- Compare (learned) Tikhonov functional approaches against several other reconstruction methods
- Low-dose computed tomography reconstruction is used as real-world application for the comparison

DIP Approaches to Inverse Problems

Recently, the Deep Image Prior (DIP)[4] has been introduced as a novel image reconstruction technique. The idea combines the architecture of generative neural networks with model-based optimization by solving the problem

$$\hat{\theta} = \arg\min_\theta \mathcal{L}(A\varphi_\theta(z), y^\delta)$$

with a variant of stochastic gradient descent (SGD) and early stopping. This yields a reconstruction

$$\hat{x} = \varphi_{\hat{\theta}}(z),$$

which is observed to be regularized in a beneficial way.

Interesting questions for further research are:

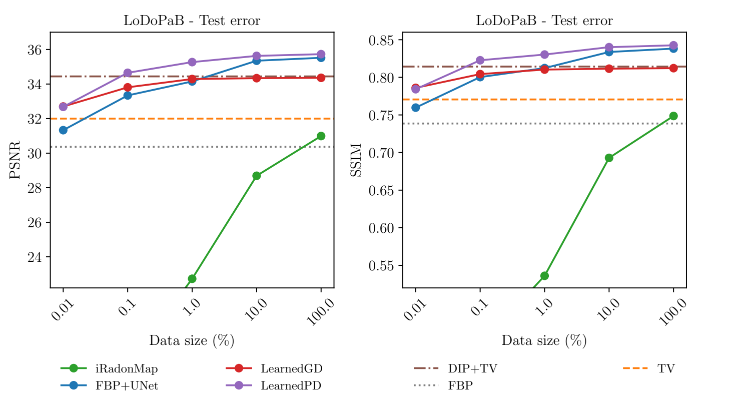
- What is essential for the regularization effect? Aspects of the architecture, the optimization strategy?
- Can the idea be improved for inverse problems involving a non-trivial forward operator (e.g. CT, MRI)?

Both of these questions involve theoretical and experimental aspects. Specific aims of the project are to

- prove mathematical properties of the method in order to provide guarantees and explain the regularizing behaviour, and to
- develop extensions of the DIP targeted at inverse problems.

Comparison on low-dose CT data

- This project is a collaboration between Daniel Otero Baguer, Johannes Leuschner and Maximilian Schmidt
- We developed an extensive dataset for the comparison of reconstruction methods called LoDoPaB-CT[3]
- We investigated the reconstruction performance and the dependency on the amount of training samples[5]
- Our approach of combining DIP and classical TV regularization was able to surpass the best methods in the low-data regime

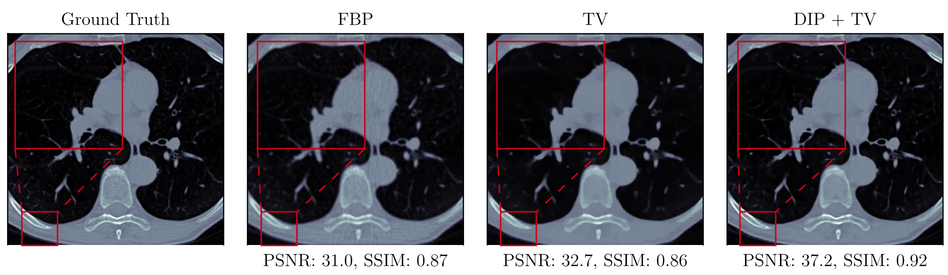


Application of Computed Tomography

- Highly relevant medical imaging application
- Linear, mildly ill-posed inverse problem
- Active field of research, including many neural network approaches

Current challenges include:

- Reconstruction from low-dose measurements
- Guarantees and explainability for network approaches
- 3D reconstruction



Unsupervised reconstruction techniques applied to a low-dose CT sample. FBP and TV are classical methods, DIP+TV is a TV-regularized DIP variant.[5]

[1] H. Li, J. Schwab, S. Antholzer and M. Haltmeier, "NETT: Solving inverse problems with deep neural networks" Inverse Problems, 2020
[2] S. Lunz, O. Öktem and C. Schönlieb, "Adversarial regularizers in inverse problems", Proceedings of the 32nd International Conference on Neural Information Processing Systems 2018, pp. 8516–8525
[3] J. Leuschner, M. Schmidt, D. O. Baguer and P. Maaß, "The LoDoPaB-CT Dataset: A Benchmark Dataset for Low-Dose CT Reconstruction Methods" *ArXiv preprint abs/1910.01113*, 2019
[4] V. Lempitsky, A. Vedaldi and D. Ulyanov, "Deep Image Prior" *2018 IEEE/CVF Conference on Computer Vision and Pattern Recognition*, Salt Lake City, UT, 2018, pp. 9446–9454, doi: 10.1109/CVPR.2018.00984
[5] D. O. Baguer, J. Leuschner and M. Schmidt, "Computed Tomography Reconstruction Using Deep Image Prior and Learned Reconstruction Methods" *ArXiv preprint abs/2003.04989*, 2020

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