

Nonnegative Matrix Factorization (NMF)

Classical NMF Problem

- For a given matrix $\mathbf{X} \in \mathbb{R}_{\geq 0}^{N \times T}$, find matrices $\mathbf{B} \in \mathbb{R}_{\geq 0}^{N \times K}$ and $\mathbf{C} \in \mathbb{R}_{\geq 0}^{K \times T}$ with $K \ll \min\{N, T\}$, s.t. $\mathbf{X} \approx \mathbf{BC}$.

Task Areas

- Used e.g. for dimension reduction, data compression, basis learning and source separation.

Optimization Theory

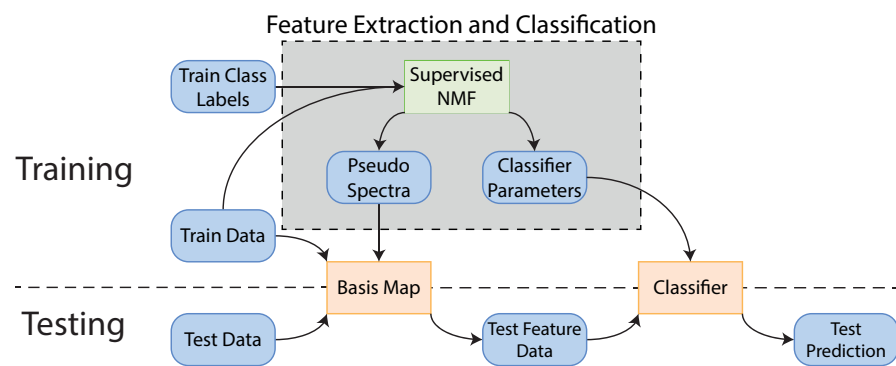
- Review and extension of the *majorize-minimization* principle for generalized NMF models.¹

$$\min_{\mathbf{B}, \mathbf{C} \geq 0} \mathcal{D}_{\text{NMF}}(\mathbf{X}, \mathbf{BC}) + \sum_{\ell=1}^L \gamma_{\ell} \mathcal{P}_{\ell}(\mathbf{B}, \mathbf{C})$$

Applications

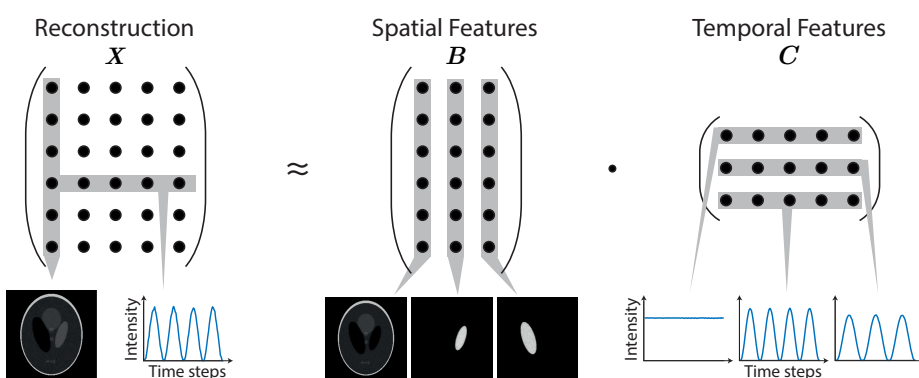
MALDI Imaging

- Development of supervised NMF models for extracting tumor-specific spectral patterns.²



Dynamic Computerized Tomography

- Reconstruction and low-rank decomposition for dynamic inverse problems via NMF.³



Clustering

- Development of an orthogonal NMF model with total variation regularization to enforce spatial coherence.
- Find connections between regularized orthogonal NMF and generalized K-means models.

Audio Source Separation

We want to identify and separate the contributions of different instruments in a music recording.



Figure 1: Musical score for a piece by Mozart

This is much easier if the tones from one instrument follow the same pitch-invariant pattern. We developed a pitch-invariant and frequency-uniform time-frequency representation that combines properties from the mel spectrogram and the constant-Q transform.⁴ However, in order to improve the resolution, we rely on a *sparse pursuit* method.⁵

Sparse Pursuit

We use an algorithm to represent a discrete mixture via shifted continuous patterns from a parametric model.

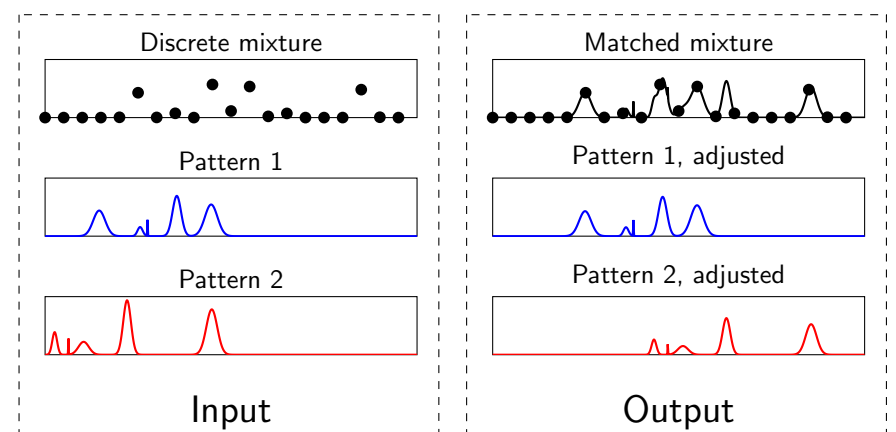


Figure 2: Test example for the sparse pursuit algorithm

With this, we can recover the spectra of the individual instruments.

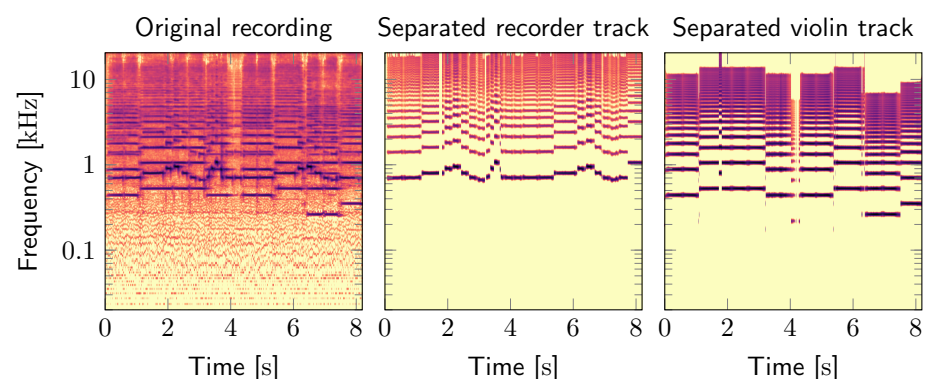


Figure 3: Separation result for the above music piece

In *blind* separation, the sounds of the instruments are not known a-priori, so we train a *dictionary* containing the relative amplitudes of the harmonics.

¹P. Fensel, P. Maass: *A survey on surrogate approaches to non-negative matrix factorization*. Vietnam Journal of Mathematics, 46 (2018), pp. 987-1021.

²J. Leuschner, M. Schmidt, P. Fensel, D. Lachmund, T. Boskamp, P. Maass: *Supervised non-negative matrix factorization methods for MALDI imaging applications*. Bioinformatics, 35 (2018), pp. 1940-1947.

³S. Arridge, P. Fensel, A. Hauptmann: *Joint reconstruction and low-rank decomposition for dynamic inverse problems*. arXiv:2005.14042, 2020.

⁴S. Schulze, E. J. King: *A frequency-uniform and pitch-invariant time-frequency representation*. PAMM 2019.

⁵S. Schulze, E. J. King: *Sparse pursuit and dictionary learning for blind source separation in polyphonic music recordings*. Submitted. arXiv:1806.00273, 2020.