

Tikhonov regularization with tolerances in the penalty

Main challenge: reconstruction of solutions that lie within intervals. Motivation is often found in:

- applications in production engineering (e.g., development of structural materials)
- parameter identification in PDE problems (e.g., box constraints or parameters given as intervals)

These intervals are modeled by a tolerance function dependent on ε . Assuming nonlinear $F : L_q(\Omega) \rightarrow V$ for bounded Ω and Hilbert space V , we propose an altered Tikhonov functional¹

$$J_{\alpha, \delta, \varepsilon}^{p, q}(u) := \|F(u) - v^\delta\|_V^p + \alpha \|u - u^*\|_{q, \varepsilon}^q \quad (1)$$

with noisy data v^δ s.t. $\|v - v^\delta\| \leq \delta$, regularization parameter $\alpha > 0$, reference solution $u^* \in L^q(\Omega)$ and $p, q \in [1, 2]$.

In the penalty we use the $L_{q, \varepsilon}$ -insensitive measure, inspired by Vapnik's ε -insensitive function², which for $u \in L_q(\Omega)$, $\Omega \subset \mathbb{R}^n$, we define as

$$\|u\|_{q, \varepsilon}^q := \int_{\Omega} (\max\{|u(x)| - \varepsilon, 0\})^q dx.$$

Example: noisy data differentiation

Consider the problem $Ku^\dagger = v$ with linear operator $K : L_2(0, 1) \rightarrow L_2(0, 1)$ defined by $Ku(x) = \int_0^x u(s) ds$. For $p, q = 2$ we compare the approximations obtained from the minimization of (1) with and without tolerance ε .

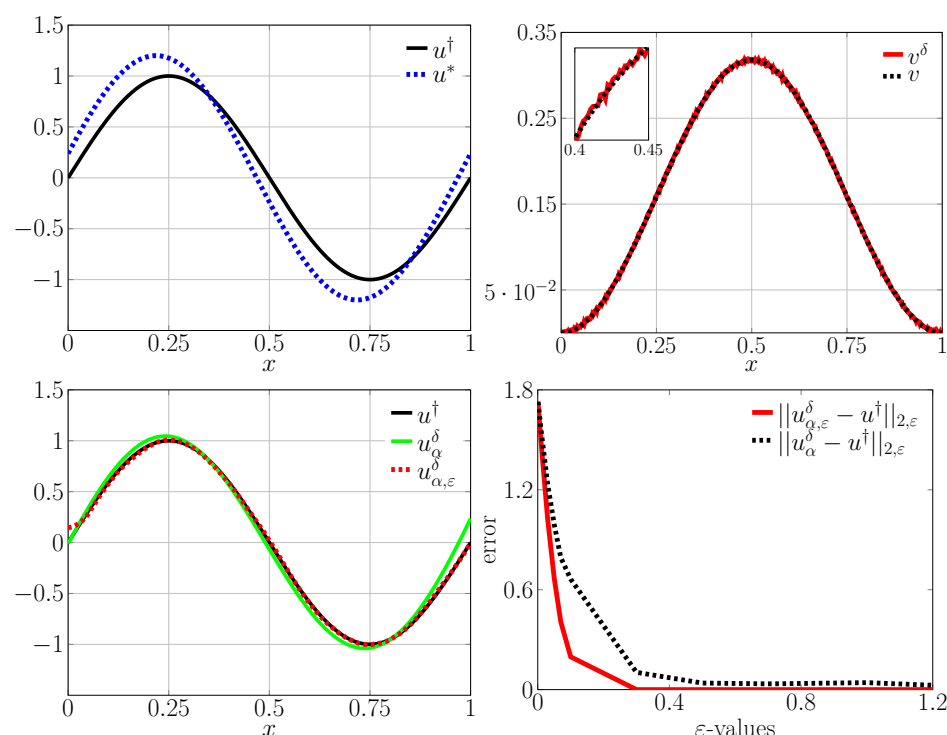


Figure 1. Ground truth $u^\dagger = \sin(2\pi x)$ and reference solution $u^* = 1.2 \sin(2\pi x + 0.2)$ (top left), and true data $v = Ku^\dagger$ and noisy data $v^\delta = v + n(\delta)$ for $\delta = 0.0015$ (top right). Comparison of minimizers u_α^δ and $u_{\alpha, \varepsilon}^\delta$ to u^\dagger for $\alpha = 0.01$, $\varepsilon = 0.3$ (bottom left) and mean approximation error calculated in the insensitive measure over 50 runs for $\varepsilon \in [0.001, 1.2]$ (bottom right).

Assisted Annotation Tool in Digital Pathology

An automatic and efficient diagnosis of tumors is a significant challenge in digital pathology.

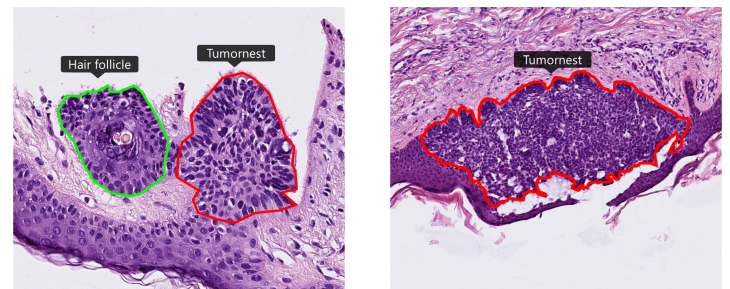
State of the Art: Modern deep learning methods have been successfully used to automatically detect tumors in histological images³.

Motivation: Creating a large and balanced dataset is a complex and time-consuming process in medical imaging.

Aim: Speed up the annotation process to make optimal use of the pathologists' limited time.

- Design a deep learning boosted method, based on convolutional neural networks (CNN) that automatically segments a structure in a bounding box.

Scientific Question: Investigate Tikhonov and tolerances regularization techniques to enhance the training of the CNN.



Segmentation by Polygons

Semiautomatic tool (initial approach): Based on bilateral filter, morphological operations, and thresholds provided by the user.

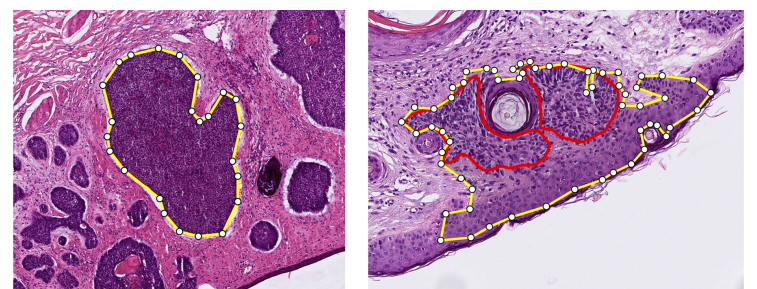


Figure 2. The classical approach works well when the area is well delimited (left image). If the color intensities are similar it does not bring good results (right image). The red polygon defines the desired region and the yellow one the obtained by the classical approach.

DL approach: Based on convolutional encoder/decoder networks for semantic segmentation⁴.

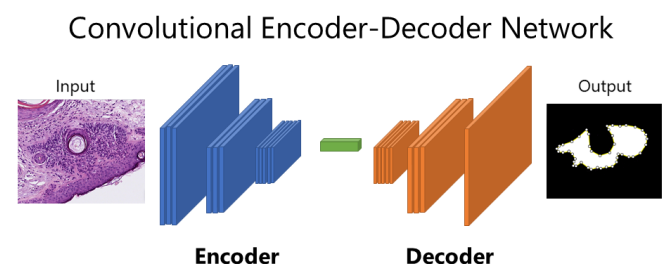


Figure 3. Baseline architecture for semantic segmentation

¹Sfakianaki, G., Piotrowska-Kurczewski, I. (in preparation, 2020). On the connection between sparsity and tolerances in parameter space.

²Cortes, C. and Vapnik, V. (1995). Support-vector networks. In Machine Learning, pages 273–279.

³Hu, Z., Tang, J., Wang, Z., Zhang, K., Zhang, L., & Sun, Q. (2018). Deep learning for image-based cancer detection and diagnosis a survey. Pattern Recognition, 83, 134–149.

⁴Ronneberger, O., Fischer, P., Brox, T. (2015). U-Net: Convolutional networks for biomedical image segmentation. In International Conference on MICCAI (pp. 234–241). Springer, Cham.