PhD projects R2

R2-3: Non-linear Dynamics and Parameter Identification

R2-7: Parameter and Structure Identification for Complex Dynamical Systems

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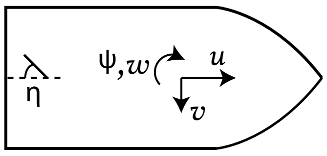
RTG 2224

Application to ship maneuvering[1]

Velocities and angles for a marine craft:

- u: surge velocity
- v: sway velocity
- w: yaw velocity
- Ψ: yaw angle
- η: thruster angle



Proportional control: $\eta(w,\psi) = \varepsilon_w w + \varepsilon_\psi \psi$

ε_w: yaw damping control; ε_ψ: yaw restoring control

Equations of motion of our model:

$$\begin{pmatrix} \dot\psi \\ \dot u \\ \dot v \\ \dot w \end{pmatrix} = \begin{pmatrix} w \\ c_0 + c_1 u + c_2 u^2 + c_3 vw + \tau_1 \cos\eta \\ c_4 v - c_5 w + c_6 uv + c_7 uw + f(v,w) + \tau_2 \sin\eta \\ c_5 v + c_4 w + c_8 uv + c_9 uw + g(v,w) + \tau_3 \sin\eta \end{pmatrix}$$

where forces at high Reynolds number and large scale entail

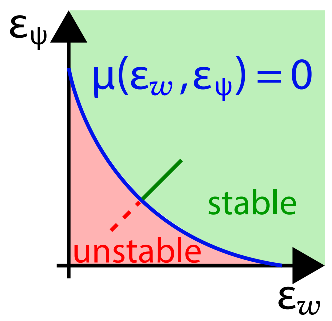
$$f(v,w) = a_{11} v|v| + a_{12} v|w| + a_{21} w|v| + a_{22} w|w|,$$
$$g(v,w) = b_{11} v|v| + b_{12} v|w| + b_{21} w|v| + b_{22} w|w|.$$

Absolute value functions come from cross-flow drag.

Analysis of non-smooth bifurcations[2]

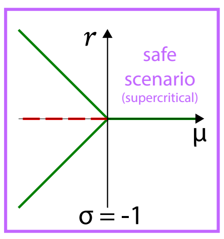
Identification of characteristic parameters that determine whether a bifurcation of periodic states is 'safe' or 'unsafe' (super- or subcritical).

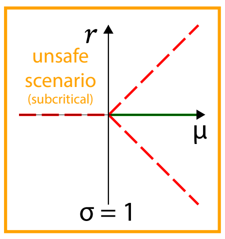
Hopf bifurcations (HB) in $(\varepsilon_w, \varepsilon_\psi)$-plane:



Radial normal form with criticality controlled by σ:

$$\dot r = -\mu r + \sigma r|r|$$





Is the HB safe or unsafe for the straight motion of the ship?

Smooth theory not applicable: new approach required!

Abstract setting: $\dot{\mathbf{u}} = A(\mu)\mathbf{u} + G(\mathbf{u}), \quad \mathbf{u} \in \mathbb{R}^n,$
$G(\mathbf{u}) = \mathcal{O}(|\mathbf{u}|^2)$, piecewise smooth.

We rigorously derive explicit generalised Lyapunov coefficients $\sigma_\#, \sigma_2, \ldots$, whose signs determine criticality and scaling.

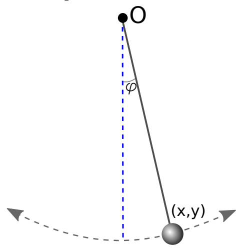
Theorem (sample):
Amplitude r_0 of periodic orbit:

- $\sigma_\# \neq 0$: $r_0 = -\frac{3\pi}{2\sigma_\#}\mu + \mathcal{O}(\mu^2)$
- $\sigma_\# = 0$: $r_0 = \sqrt{\frac{2\pi\omega}{\sigma_2}\mu} + \mathcal{O}(\mu)$

For the 'Hamburg test case' ship we prove: all bifurcations are safe!

Structure Identification for Hamiltonian Systems

Simple Pendulum



Hamiltonian mechanics:
$H(\varphi,\omega) = \frac{3}{2}\cdot\omega^2 + 5\cdot\cos(\varphi)$

Differential equation system:

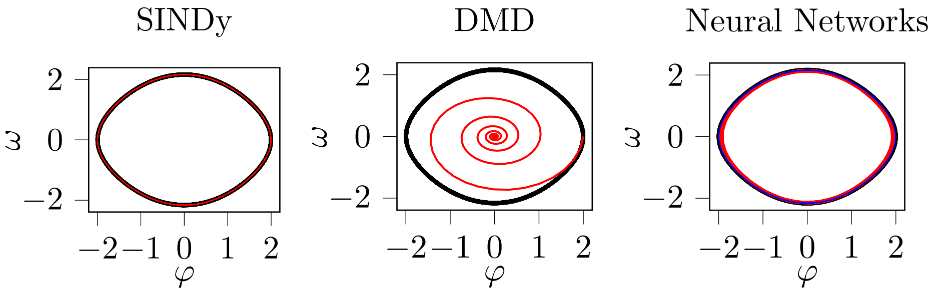
$$\dot X = \begin{pmatrix} \dot\varphi \\ \dot\omega \end{pmatrix} = \begin{pmatrix} \partial H/\partial\omega \\ \partial H/\partial\varphi \end{pmatrix} = \begin{pmatrix} 3\cdot\omega \\ -5\cdot\sin(\varphi) \end{pmatrix}$$

Data: $\left[X, \dot X\right]$

Training: 50 random initial position round $X = [2.0, 0.0]$, solved for 3 s (RK4), added noise

Test: Initial position $X = [2.0, 0.0]$ solved for 30 s (RK4)

Comparison of Different Identification Approaches



Solved with RK4: Black: Exact ode solved, Red: SINDy/DMD/Base model, Blue: HNN model

Sparse Identification of Nonlinear Dynamics (SINDy)[3]

- Library: $\Theta(X) = [1 \;\; X \;\; X^{P_1} \;\; \cdots \;\; \sin(X) \;\; \cos(X) \;\; \cdots]$
- Solve linear System $\dot X = \Theta(X)\,\Xi$
- $\Rightarrow \Xi$ shows which base-functions are used to build ODE

Application:

- Polynomial degree: 3, use trigonometric functions
- Learned model: $\begin{pmatrix} \dot\varphi \\ \dot\omega \end{pmatrix} = \begin{pmatrix} 3.0001\cdot\omega \\ -4.99995\cdot\sin(\varphi) \end{pmatrix}$

Dynamic Mode Decomposition[4]

- Approximation of the modes of the Koopman operator
- Extracts temporal features,
- Used for state estimation and prediction.

Application:

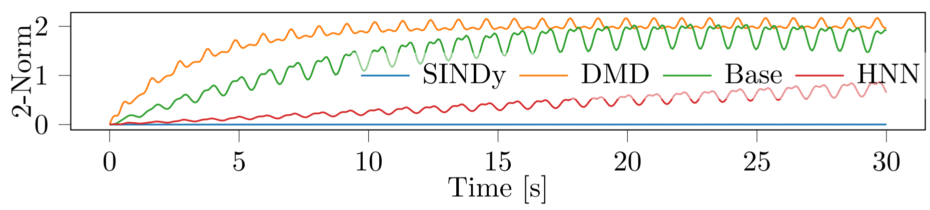
- Looses energy during prediction

Neural Networks

- Base-Net learns $X \mapsto \dot X$
- Hamiltonian Net[5] learns $X \mapsto H_\theta(X)$,
  Loss: $\mathcal{L} = \|\partial H_\theta/\partial\omega - \dot\varphi\|^2 + \|\partial H_\theta/\partial\varphi - \dot\omega\|^2$

Application:

- Base-Net (red): Slowly losing energy
- HNN (blue): Preserve's Energy



[1] Steinherr, M., & Rademacher, J. Nonlinear effects of stabilising ship motion with P-control. In preparation.
[2] Steinherr, M., & Rademacher, J. (2020). Lyapunov coefficients for Hopf bifurcations in systems with piecewise smooth nonlinearity. Submitted.
[3] Brunton, S.L., Proctor, J.L., & Kutz, J.N. (2016). Discovering governing equations from data by sparse identification of nonlinear dynamical systems. In PNAS (Vol. 113, No. 15, p. 3932–3937)
[4] Proctor, J.L., Brunton, S.L., & Kutz, J.N. (2016). Dynamic Mode Decomposition with Control. In SIAM Journal on Applied Dynamical Systems (Vol. 15, No. 1, p. 142-161)
[5] Greydanus, S., Dzamba, M., & Yosinski, J. (2019). Hamiltonian Neural Networks. In Advances in Neural Information Processing Systems (Vol. 32, p. 15379–15389)

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