

The Concept of Function in the Transition to Higher Secondary Education

Need for Action

Teaching and learning Mathematics in the transition to higher secondary education (usually 10th to 11th grade) is often seen as problematic. There are various possible explanations for this:

- Mathematical education in lower secondary school does not specifically prepare students for higher secondary education.
- The learning group is very heterogeneous.
- Dealing with functions in lower secondary education is not known (probably orientated at the use).
- The learning groups have a lack of broad knowledge about the structural properties of the function concept.

According to the *Bremer Bildungsplan*, students at higher secondary school have to integrate or differentiate functions, invent new functions for modeling and solving problems, handle typical properties and so on.

The change will be implemented and realized through epistemic processes. The change will be ...

- ...from a use-orientated to a more flexible and substantiated approach.
- ...from a fragmented to a more integrated concept.
- ...based on the types of functions and the representations.

Design-Context

Anthropological Theory of Didactic (ATD; Chevallard & Sensevy 2014)

- Investigation of mathematical and didactic activities in a given institutional setting
- Praxeologies: Types of tasks, a set of techniques (ways of doing), technology (discourse of the technique) and theory (basis and support for the technological discourse)

RBC+C Model (Schwarz et al. 2009)

- Investigation of individual epistemic processes
- RBC+C: recognizing, building-with, constructing and consolidation

GCSt Model (Bikner-Ahsbahr und Halverscheid 2014)

- Investigation of collective epistemic processes
- GCSt: gathering, connecting and structure seeing

Functions (Beckmann 2007)

- An important but difficult mathematical concept
- High complexity: related to the content and the representations
- Studies have shown: Students often have only constructed a limited knowledge about functions

Conceptual Blending (Fauconnier and Turner 2003)

- „A basic mental operation that leads to new meaning, global insight and conceptual compression“
- A network model that combines four spaces: A generic space (constituted by commonalities of the input spaces), two partially matched input spaces, and a blended space (the new construction)

Research Questions

- How can the conceptual change of functions be successfully designed and maintained?
- Which conditions facilitate or complicate the transformation process?
- When and in which form do problems emerge?

Design-Subject

The approach to functions requires the integration of different aspects (relation, covariance and object; cf. Vollrath 1989, pp. 8-16). Representations have to be interpreted according to these aspects. Types of functions are shaped by their structural properties. The structural properties could bundle different types of functions or could only be true for one type of functions. Thereby, overlapping properties are less concrete.

Design-Conception

The core idea of **task construction** is interpreting known geometrical formulas as a function (cf. Malle 1993, pp. 267-270). For developing the learning and teaching arrangement, the GCSt Model will be used to generate epistemic processes.

The posed **hypotheses** are related to the epistemic processes and to the traces of limited praxeologies.

The **principle of design** is: „To construct new knowledge about functions, the conceptual blending in combination with different representations as well as transference between different representations, could be used.“

The following learning goals are linked: The students should ...

- ...learn to talk about the same functions in different representations.
- ...recognize similarities and differences between linear and quadratic functions (concerning the functional aspects and visible properties of representations), while they switch between different types of functions.
- ...construct knowledge, which initiates changes.

Design-Testing

The analysis will be about reconstructing habitual practices of functions in the sense of ATD as well as the epistemic processes (related to the change) with the aid of the RBC+C Model and GCSt Model.

Phase 1 (videography and audiography, artefact analysis)

- Interviews with teachers (not DBR)
- Iteratively implemented interviews with students (DBR)

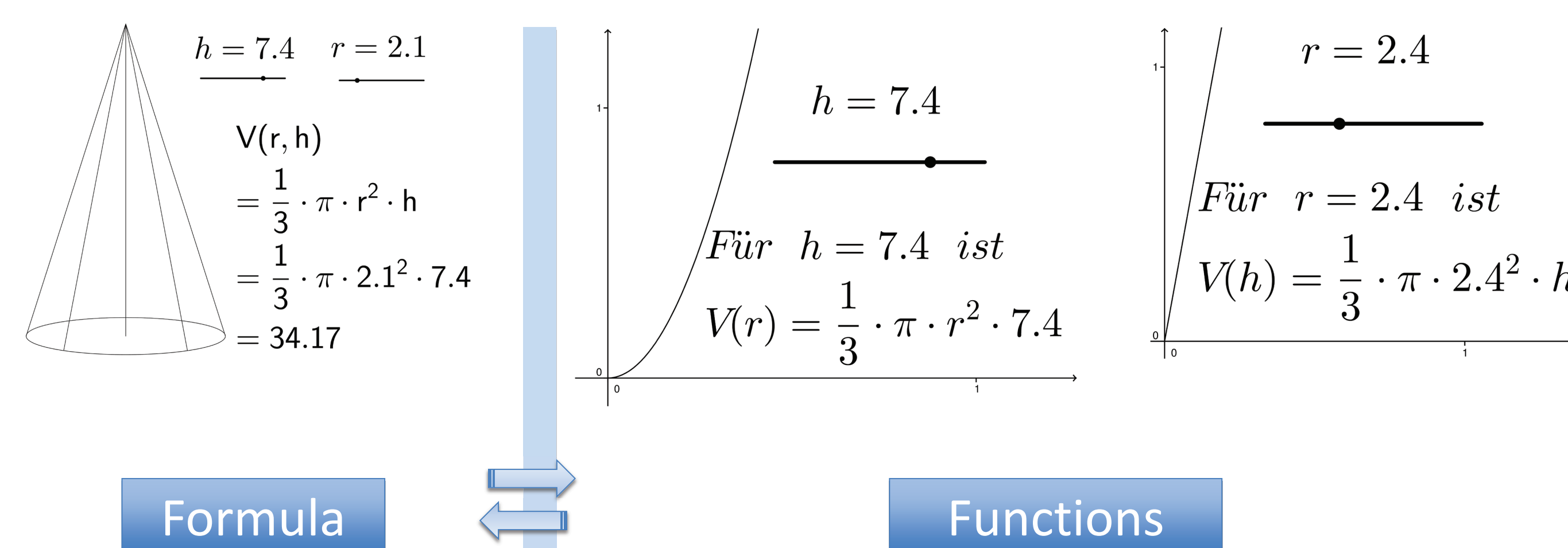
Phase 2 (videography, artefact analysis)

- Realizing the learning and teaching arrangement (implemented by a teacher)

Design-Theory

The expected design theory will be knowledge about ...

- ... which traces of praxeologies exist at the beginning of higher secondary education and in which way they complicate the epistemic processes,
- ...conditions, which facilitate or complicate the change in this transition,
- ...the change of praxeological equipment,
- ...principles for designing change,
- ... and a ...
- ...tested arrangement for fostering students in the transition to higher secondary education.



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